Ant Colony Optimization

Nuno Antunes Ribeiro

Assistant Professor

SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN | Engineering Systems and Design

Ant Colony

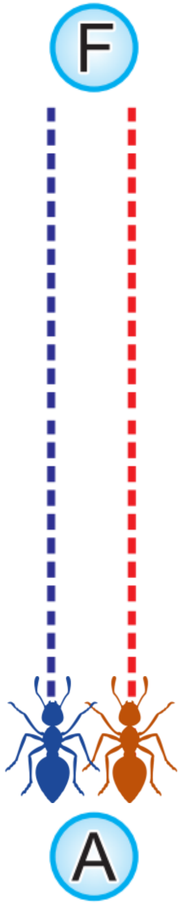
- Ants are behaviorally unsophisticated, but collectively they can perform complex tasks
- The term "ant colony" describes not only the physical structure in which ants live, but also the social rules by which ants organize themselves and the work they do.
  - Ants are continuously looking for **food**.
  - When food is found, ants return to the colony
  - Ants lay down **pheromone** whenever food is found
  - Paths with more pheromone are more likely to be followed by other ants
  - These are often the **shortest paths**
  - Many combinatorial problems can be considered as finding the shortest path on a graph.

Stigmergy

- Fundamental observation: **Stigmergy** is a form of indirect communication and coordination in which agents modify the environment to pass information to their peers

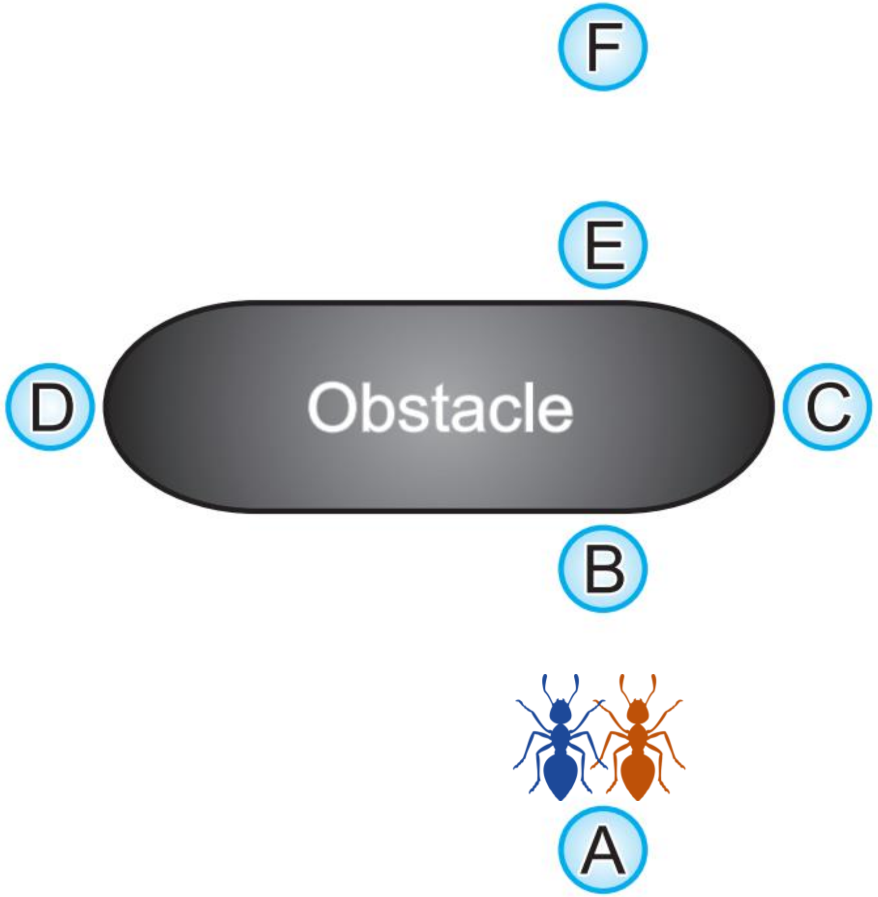
Ant Path Finding

- Ant nest (A) ; food source (F)



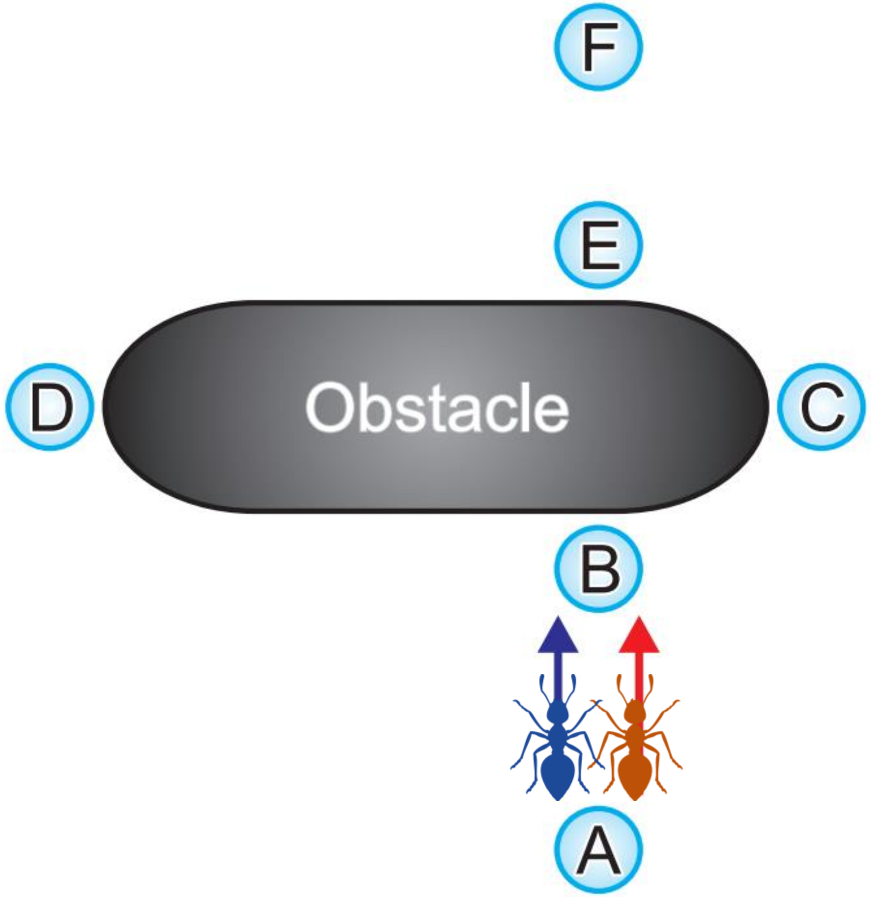
Ant Path Finding

- Ant nest (A) separated from food source (F) by obstacle



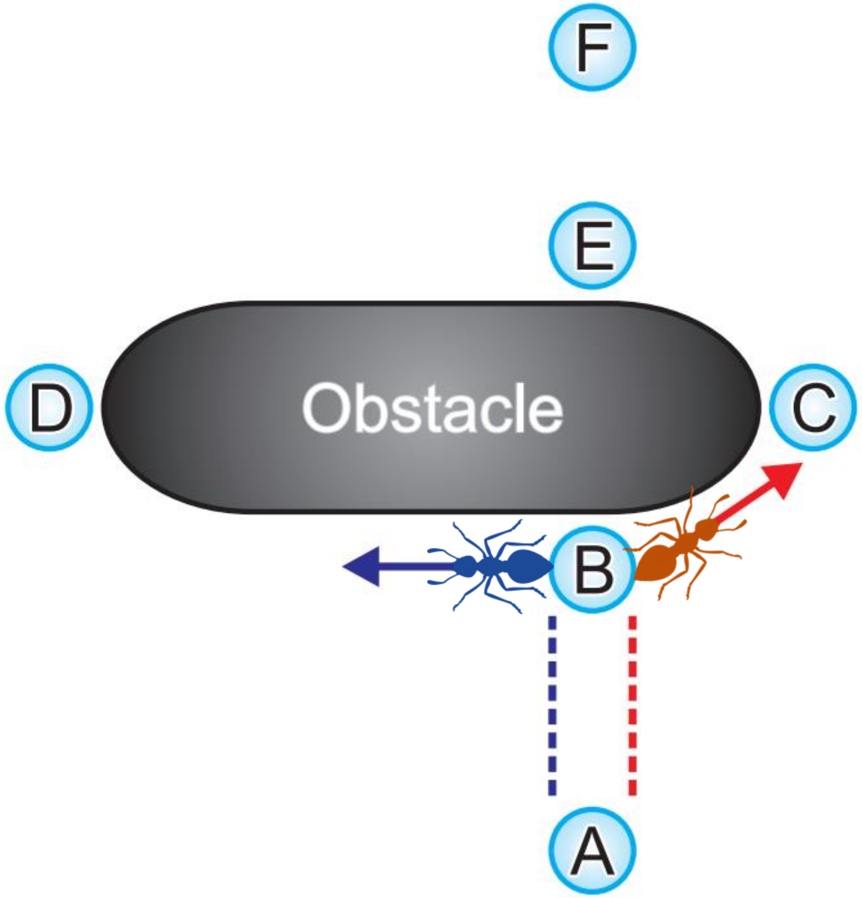
Ant Path Finding

- two ants (red and blue) leave the nest at the same time



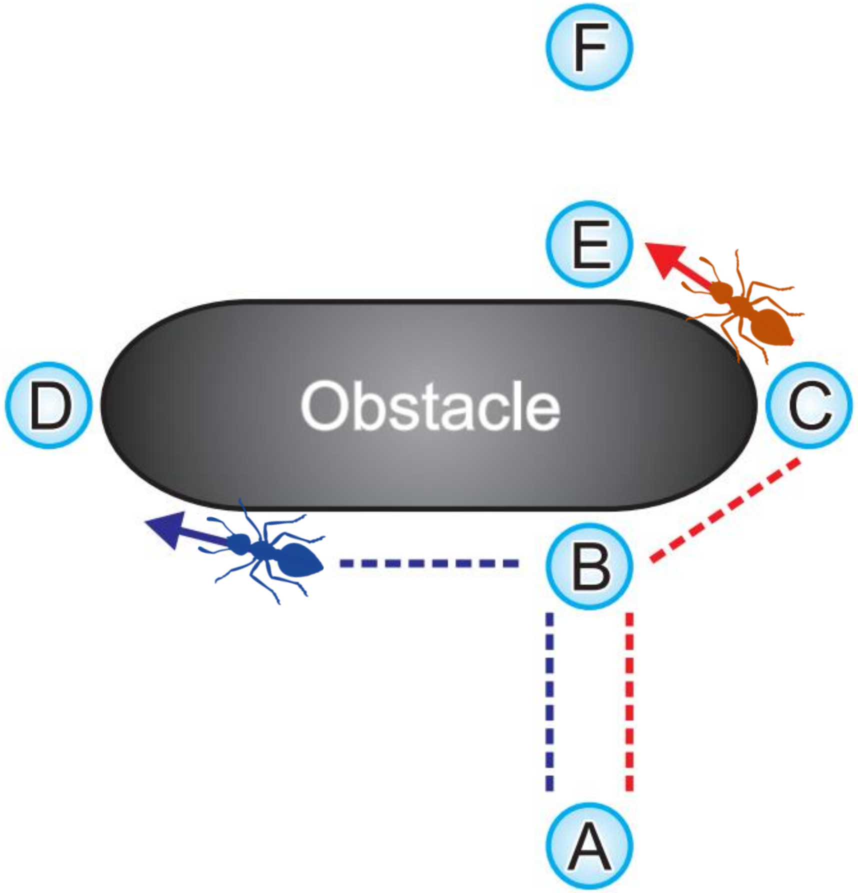
Ant Path Finding

- at the crossroad, one turns left and the other one right



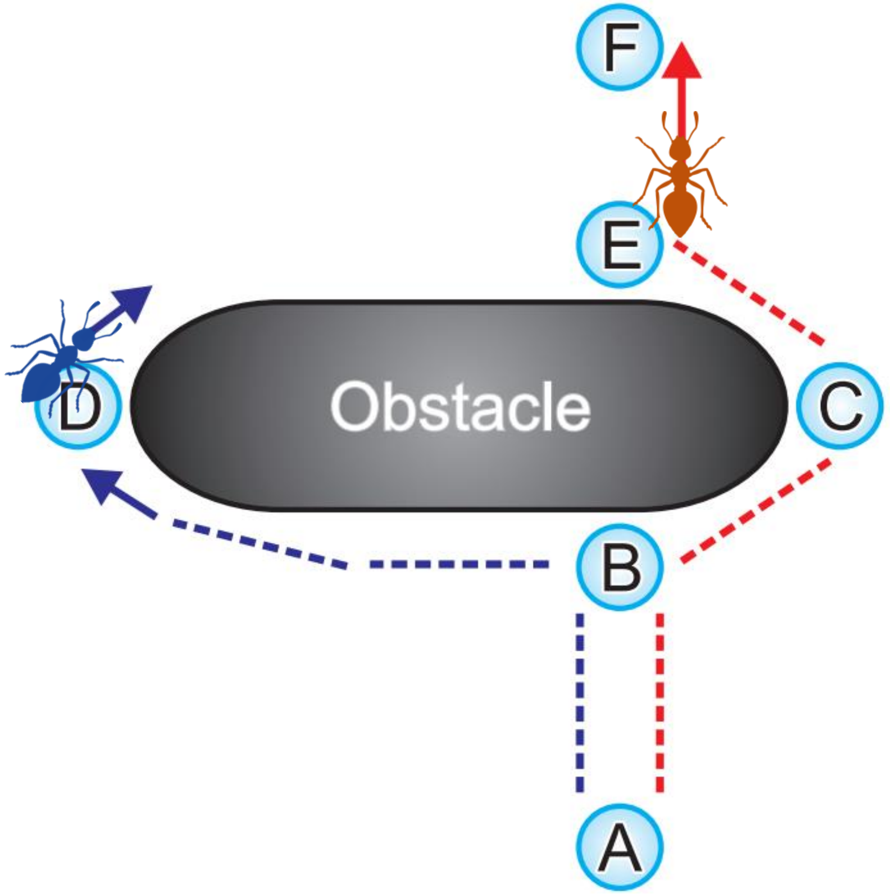
Ant Path Finding

- when moving, ants leave pheromone behind (dotted lines)



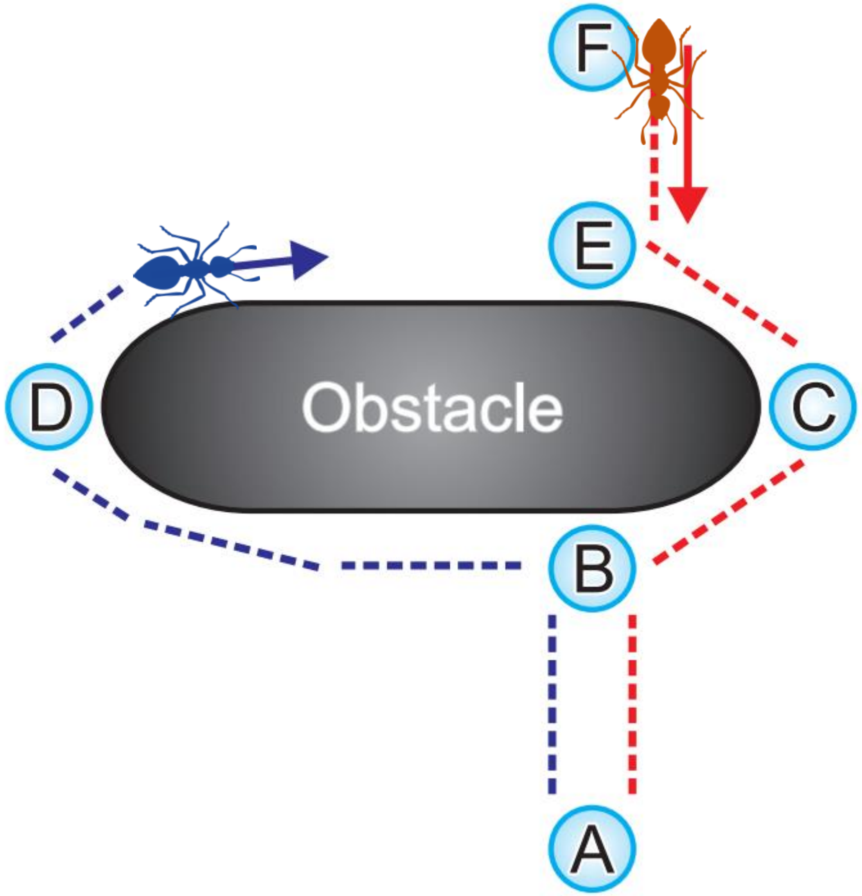
Ant Path Finding

- the one with the shorter path arrives at the food source first



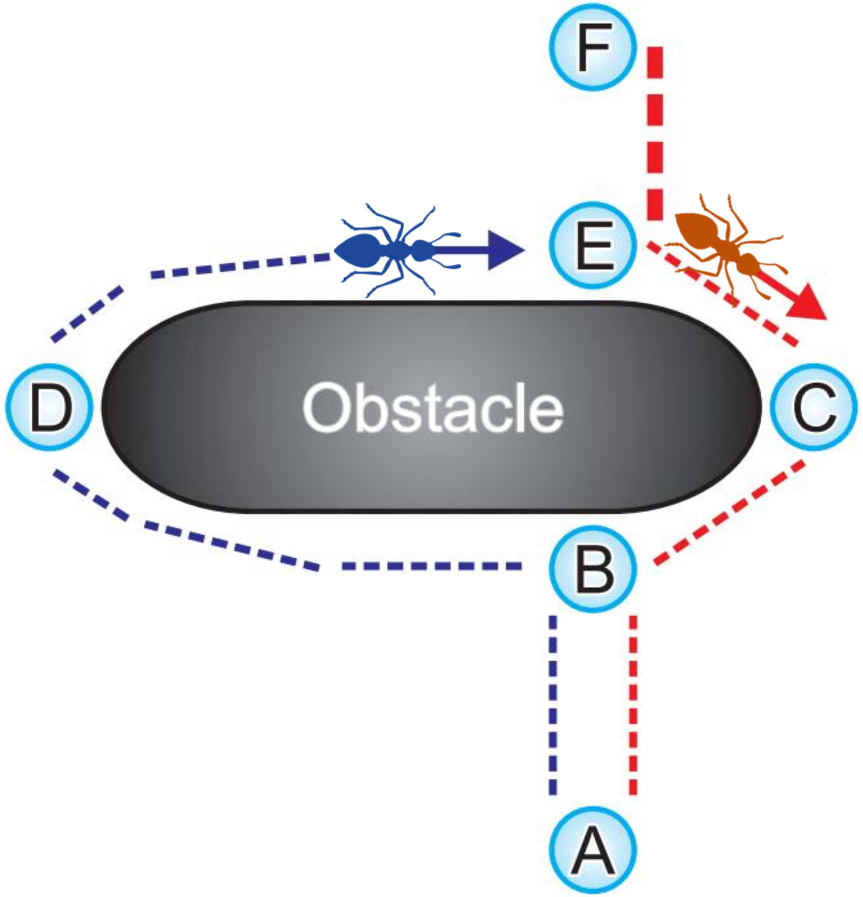
Ant Path Finding

- when it turns back, it finds pheromone on one path and follows it



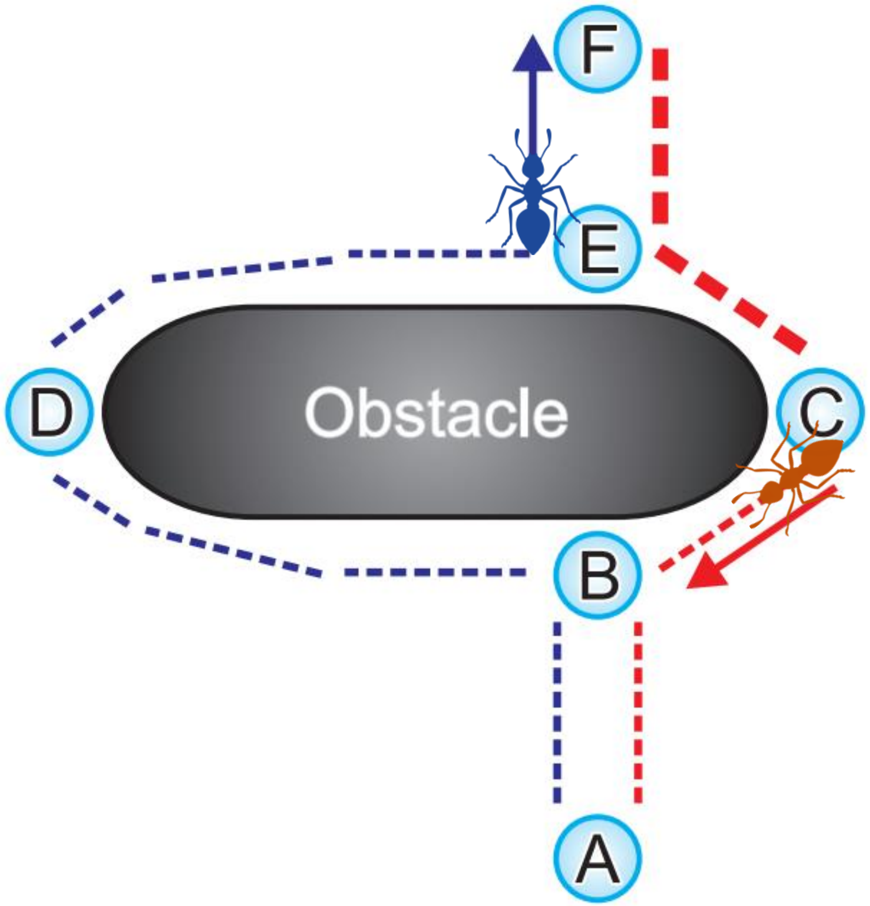
Ant Path Finding

- by doing so, it leaves even more pheromone on the path



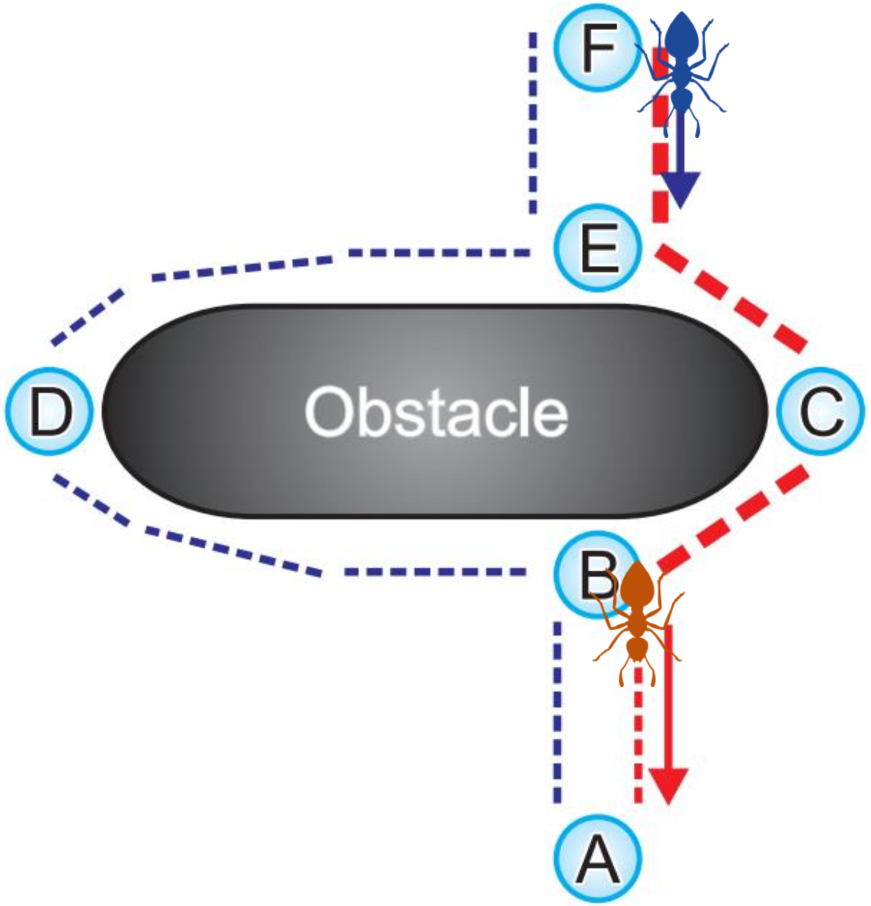
Ant Path Finding

- now the second ant arrives at the food source



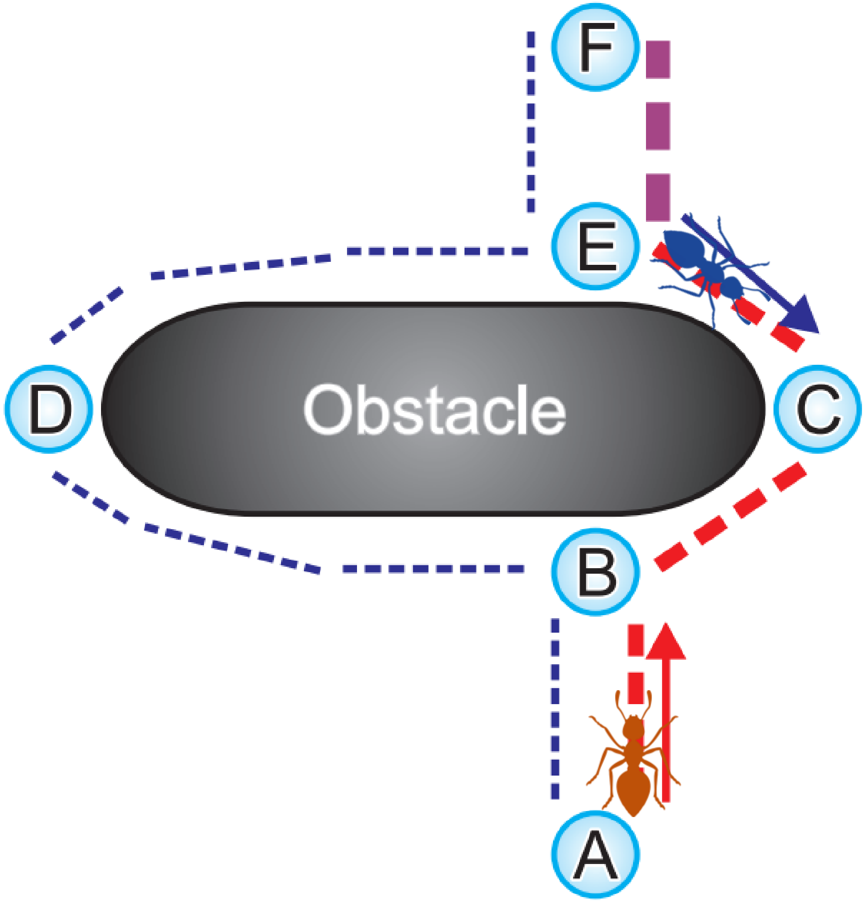
Ant Path Finding

- when it turns back, there is pheromone on both paths – but more on the red one



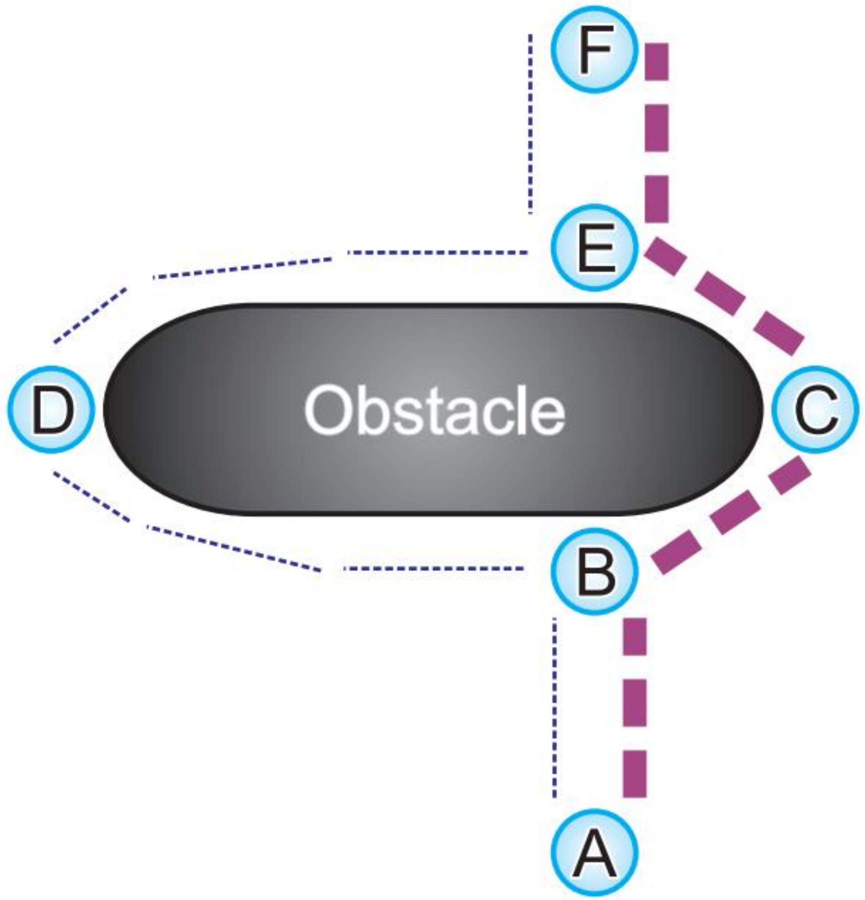
Ant Path Finding

- the pheromone on the short path gets more and more



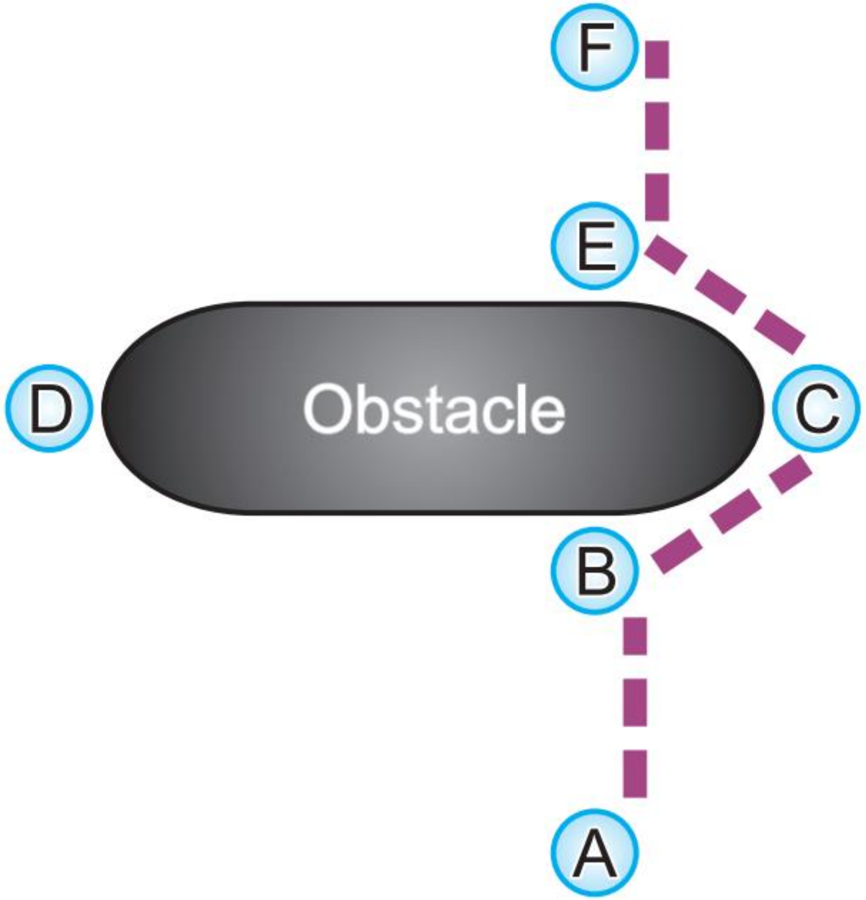
Ant Path Finding

- while the one on the blue path evaporates



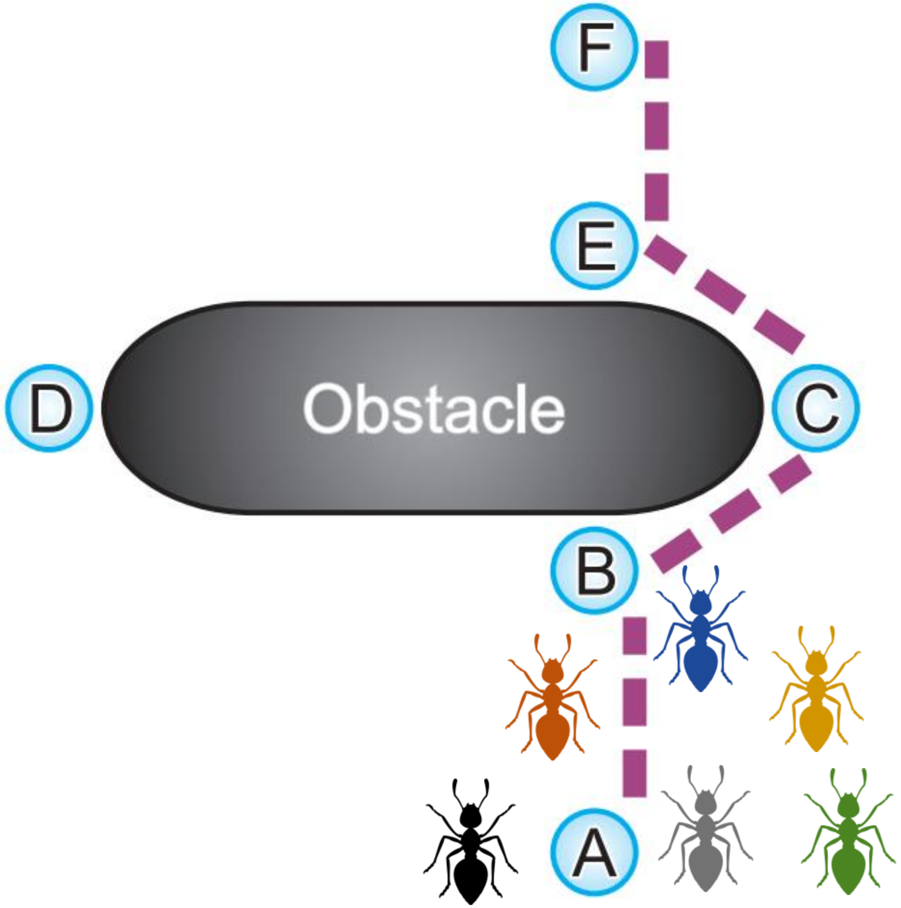
Ant Path Finding

- until only the short path has pheromone. . .



Ant Path Finding

- More paths – more ants will ensure convergence to shortest path. . .



Ant Colony Optimization

- Dorigo et. Al. (1996) have the idea to use an Ant Path simulation to solve optimization problems which can be represented as graphs – Ant Colony Optimization (ACO)
- Ants are **agents** that:
  - Move along between nodes in a **graph**.
  - They choose where to go based on **pheromone** strength
  - An ant's **path** represents a specific **candidate solution**.
  - When an ant has finished a solution, pheromone is laid on its path, according to quality of solution.
  - This pheromone trail affects the behavior of other ants by stigmergy

ACO on a graph

- A graph $G = (V, E)$ consists of a set of **vertices** (nodes) $v \in V$ and **edges** $e \in E$, with $E \subseteq V \times V$
- ACO has been designed for problems where we want to find paths through such graphs G
- ACO has three main components:
  - (simulated) ants which move through a graph along edges. The **path such an ant took represents a solution**.
  - Ants leave pheromones τ on the edges they travel along. This **pheromone helps future ants to decide which path to take**.
  - Pheromone **disappears over time** (evaporation).
- Knowledge about the problem may be incorporated which tells the ant how interesting a given edge is (η).
- Together with the pheromones, η helps the ant to decide where to go. They don't change over time.

Edge Selection - Probability

- Let us assume that all nodes i and $j \in V$ are connected with edges, i.e., we have a complete graph topology
- An ant located in node i in ACO chooses the next node j where it will go according to:
  - the **amount of pheromone** on the edge connecting i and j and
  - the **cost** (distance) of moving from i to j

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

$p_{i,j}$	probability of an ant to go to j if at location i	$\tau_{i,j}$	amount of pheromone on the edge connecting i and j
α, β	weight parameters	$\eta_{i,j}$	visibility of node j from i: inversely proportional to distance between j and i

Amount of Pheromone

- At the end of each algorithm round, "**pheromone" is dispersed** and the trails are updated (η_{ij} stays constant)

$$\tau_{i,j} = (1-\rho)\tau_{i,j} + \Delta\tau_{i,j}$$

ρ is the evaporation coefficient (fraction of pheromone disappearing into thin air)

$\Delta\tau_{i,j}$ is the amount of new pheromone dispersed

- The **amount of new pheromone dispersed is proportional to the objective value** of the path

$$\Delta\tau_{i,j} = \begin{cases} \frac{1}{f(x)} & \text{if tour } x \text{ contains edge } \overline{ij} \\ 0 & \text{otherwise} \end{cases}$$

$\Delta\tau_{i,j}$ pheromone amount to be dispersed on the edge $\overline{ij}$

Solving the TSP using ACO

Nuno Antunes Ribeiro

Assistant Professor

SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN | Engineering Systems and Design

Solving the TSP

- ACO was originally proposed to solve the traveling salesman problem (TSP)
- TSP is a graph problem by default
- We look for a path that visits all n nodes in a graph
- Basic idea
  - The cities are connected with edges
  - We have n ants
  - Each ant moves from one city to one of the cities it has not seen yet based on a given probability
  - This probability depends on the pheromones on the edges and the distances to the cities
  - Afyer all ants have completed their tout, pheromones are updated

Solving the TSP - Steps

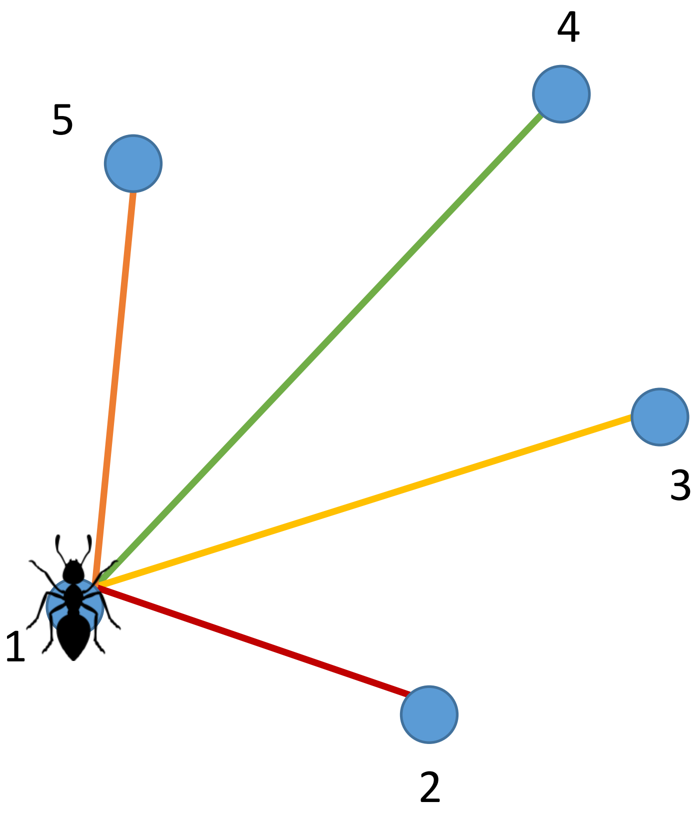
1. For each ant k of the n ants:
   1. Place ant k at a randomly chosen city/node i
   2. For $I - 1$ cities:
      1. Choose next city j from the set of cities I not yet visited by the ant (where i is its current location)
      2. City j has probability p_{ij} to be chosen as next city
   3. Return: tour x_k
2. Calculate pheromone amount $\Delta\tau_{ij}$ to be dispersed on the edge ij given the fitness of tour x_k
3. Update pheromone value τ_{ij}

Repeat until termination criteria

TSP Example

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

Iteration 1 – Ant 1



$p_{i,j}$	probability of an ant to go to j if at location i
α, β	weight parameters
$\tau_{i,j}$	amount of pheromone on the edge connecting i and j
$\eta_{i,j}$	visibility of node j from i: inversely proportional to distance between j and i

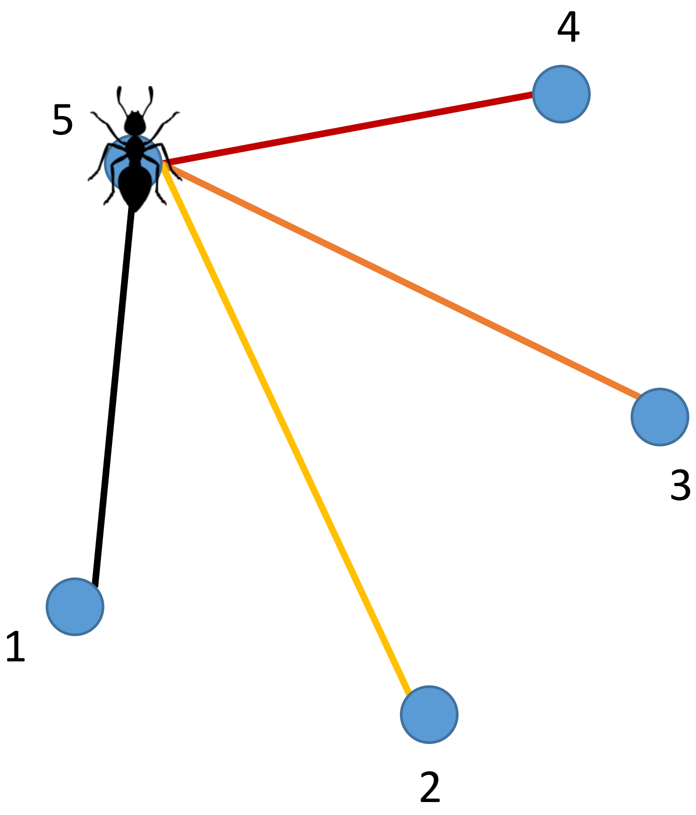
In the first iteration no pheromones are in the path
Ant selection of the next city to visit depends only on the distance
Randomized greedy approach (recall: lecture 6 - nearest neighbour heuristic)

Higher probability to select 2. Lower probability to select 3

Randomly select 1 city to locate 1 ant

TSP Example

Iteration 1 – Ant 1



Pheromones → $(\tau_{i,j})^{\alpha}$ Distance → $(\eta_{i,j})^{\beta}$

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

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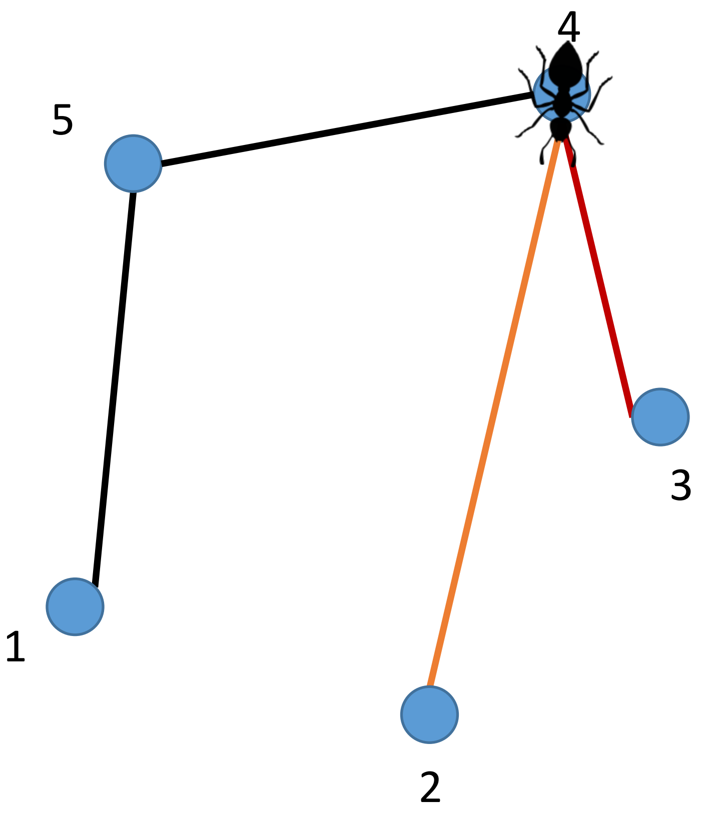
Higher probability to select 4. Lower probability to select 2

TSP Example

Pheromones **Distance**

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

Iteration 1 – Ant 1



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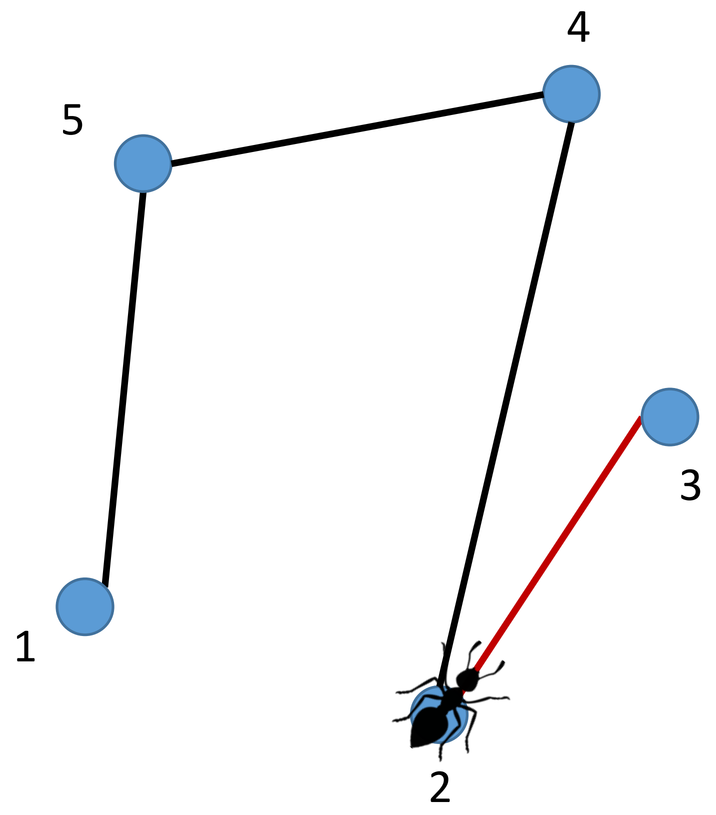
Randomized greedy approach (recall: lecture 6 - nearest neighbour heuristic)

Higher probability to select 3. Lower probability to select 2

TSP Example

Iteration 1 – Ant 1



Pheromones → $(\tau_{i,j})^{\alpha}$; Distance → $(\eta_{i,j})^{\beta}$

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Ant selection of the next city to visit depends only on the distance
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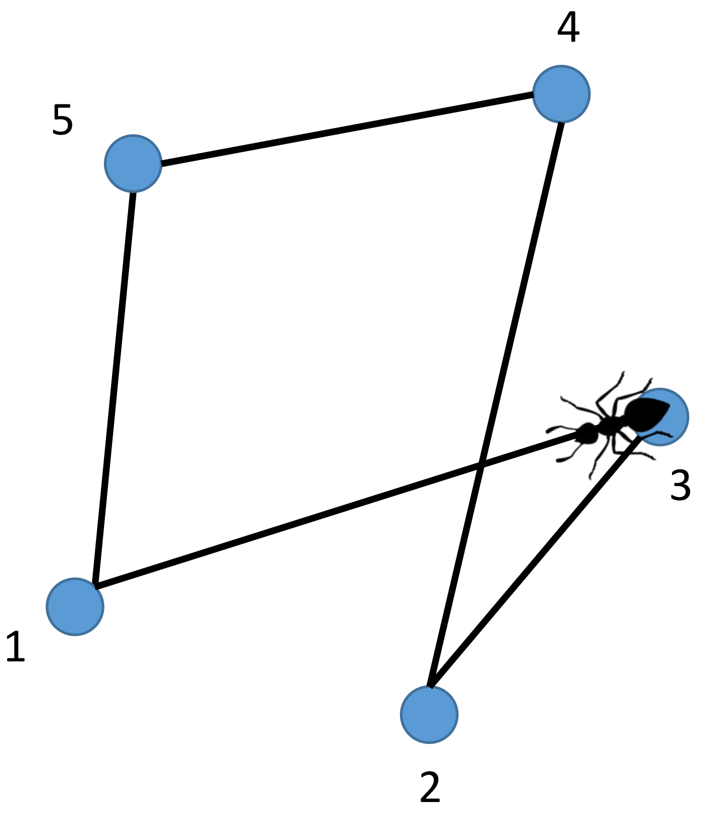
Select 3

TSP Example

Iteration 1 – Ant 1



Pheromones **Distance**

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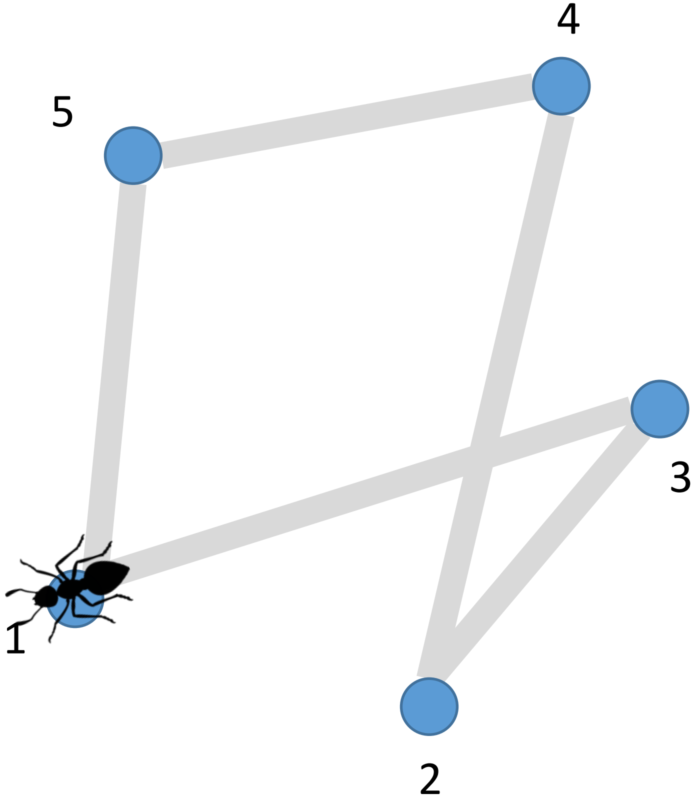
In the first iteration no pheromones are in the path
Ant selection of the next city to visit depends only on the distance
Randomized greedy approach (recall: lecture 6 - nearest neighbour heuristic)

Compute Objective Value of the tour generated (e.g. $f(x) = 20$)

TSP Example

Iteration 1 – Ant 1



- The amount of new pheromone dispersed is proportional to the objective value of the path

$$\Delta\tau_{i,j} = \begin{cases} \frac{1}{f(x)} & \text{if tour } x \text{ contains edge } \overline{ij} \\ 0 & \text{otherwise} \end{cases}$$

$\Delta\tau_{i,j}$ pheromone amount to be dispersed on the edge $\overline{ij}$

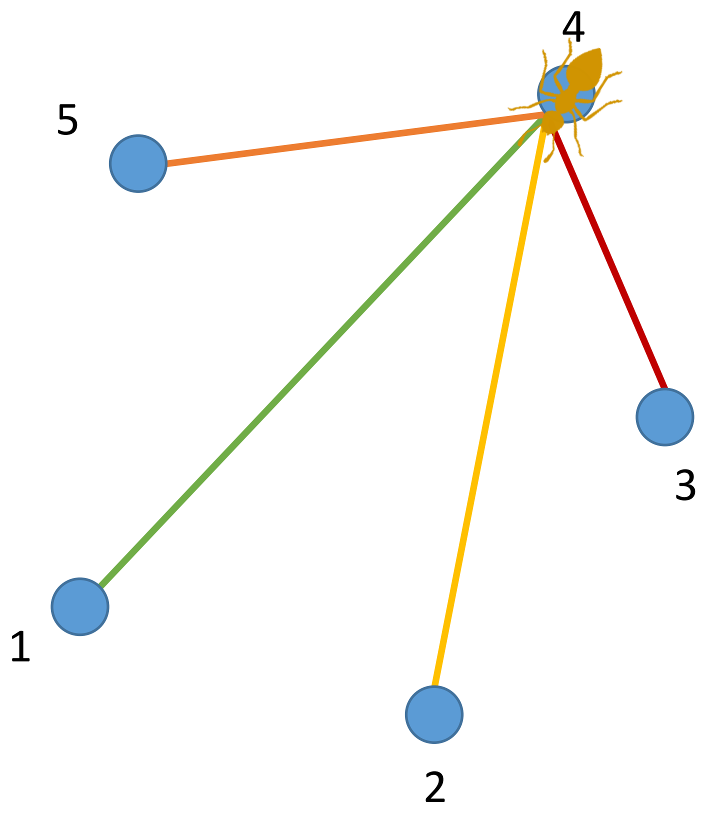
Compute Objective Value of the tour generated (e.g. $f(x) = 20$)

Compute the amount of pheromone to disperse in the path (e.g. $\Delta\tau_{ij} = \frac{1}{20} = 0.05$)

TSP Example

Pheromones Distance

Iteration 1 – Ant 2



$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

$p_{i,j}$ probability of an ant to go to j if at location i

α, β weight parameters

$\tau_{i,j}$ amount of pheromone on the edge connecting i and j

$\eta_{i,j}$ visibility of node j from i: inversely proportional to distance between j and i

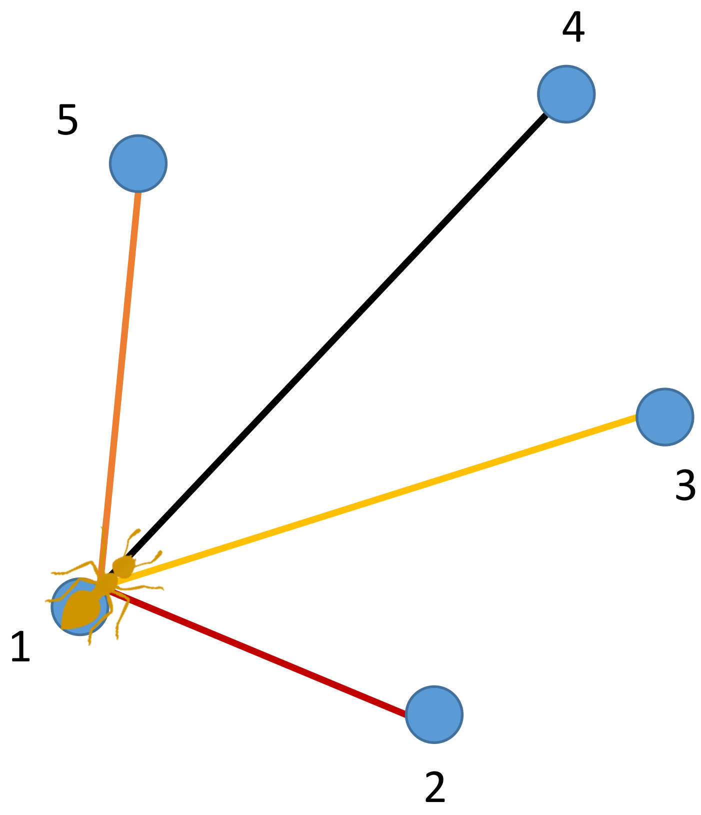
In the first iteration no pheromones are in the path
Ant selection of the next city to visit depends only on the distance
Randomized greedy approach (recall: lecture 6 - nearest neighbour heuristic)

Higher probability to select 3. Lower probability to select 1

Randomly select 1 city to locate 1 ant

TSP Example

Iteration 1 – Ant 2



Pheromones → $(\tau_{i,j})^{\alpha}$ **Distance** → $(\eta_{i,j})^{\beta}$

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

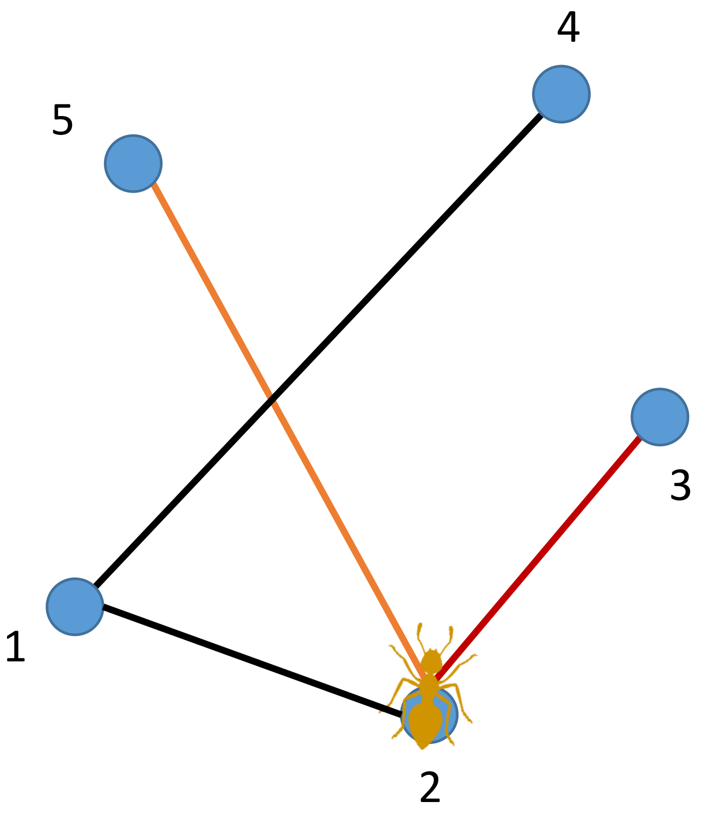
- $p_{i,j}$ — probability of an ant to go to j if at location i
- $\alpha,\ \beta$ — weight parameters
- $\tau_{i,j}$ — amount of pheromone on the edge connecting i and j
- $\eta_{i,j}$ — visibility of node j from i: inversely proportional to distance between j and i

In the first iteration no pheromones are in the path
Ant selection of the next city to visit depends only on the distance
Randomized greedy approach (recall: lecture 6 - nearest neighbour heuristic)

Higher probability to select 2. Lower probability to select 3

TSP Example

Iteration 1 – Ant 2



Pheromones → $(\tau_{i,j})^{\alpha}$ **Distance** → $(\eta_{i,j})^{\beta}$

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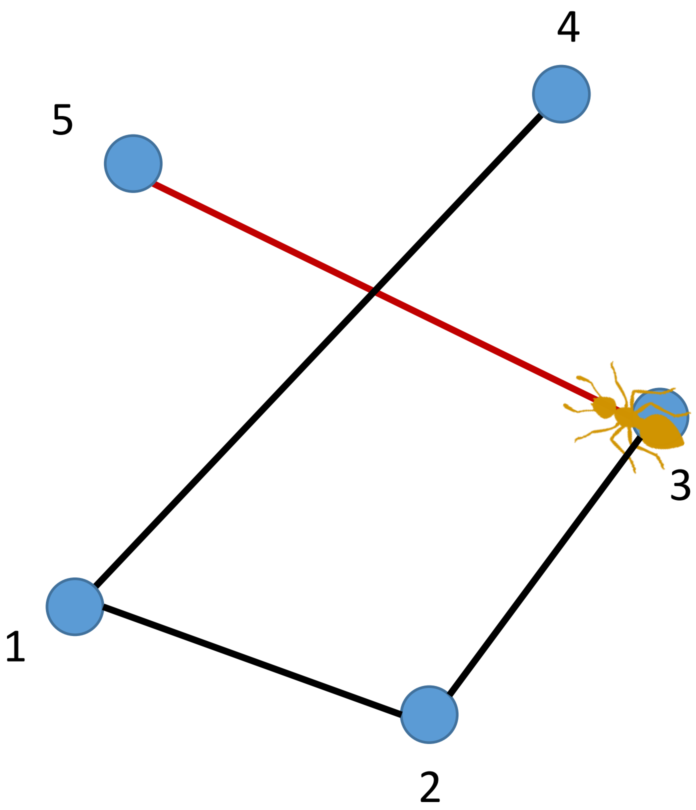
In the first iteration no pheromones are in the path
Ant selection of the next city to visit depends only on the distance
Randomized greedy approach (recall: lecture 6 - nearest neighbour heuristic)

Higher probability to select 3. Lower probability to select 5

TSP Example

Iteration 1 – Ant 2



Pheromones **Distance**

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

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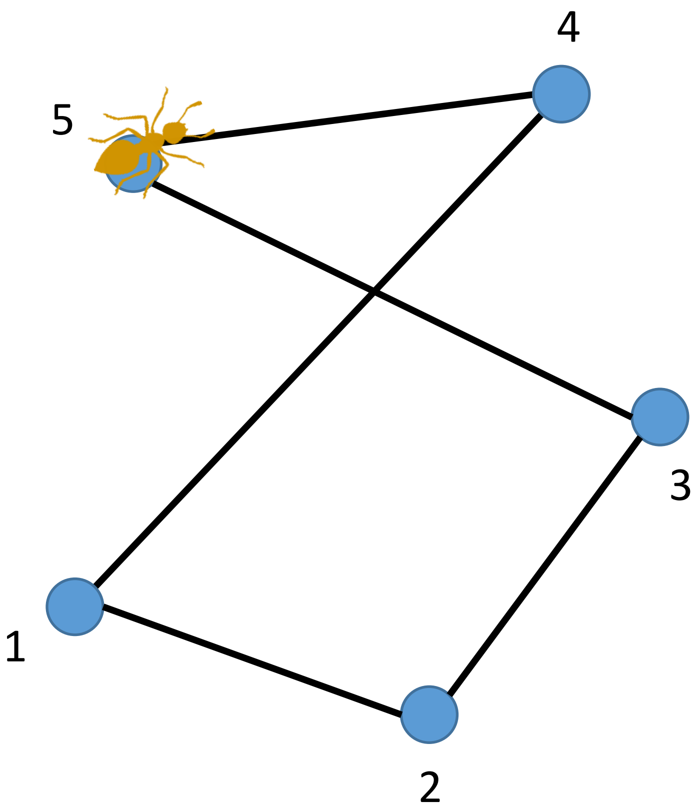
In the first iteration no pheromones are in the path
Ant selection of the next city to visit depends only on the distance
Randomized greedy approach (recall: lecture 6 - nearest neighbour heuristic)

Select 5

TSP Example

Iteration 1 – Ant 2



Pheromones — Distance

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

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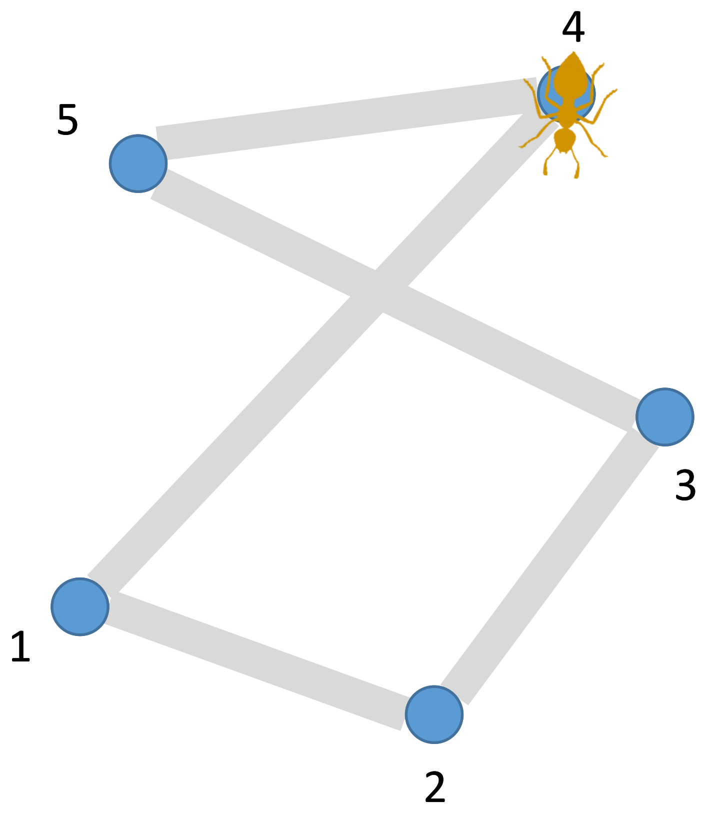
In the first iteration no pheromones are in the path
Ant selection of the next city to visit depends only on the distance
Randomized greedy approach (recall: lecture 6 - nearest neighbour heuristic)

Compute Objective Value of the tour generated (e.g. $f(x) = 15$)

TSP Example

Iteration 1 – Ant 1



- The amount of new pheromone dispersed is proportional to the objective value of the path

$$\Delta\tau_{i,j} = \begin{cases} \frac{1}{f(x)} & \text{if tour } x \text{ contains edge } \overline{ij} \\ 0 & \text{otherwise} \end{cases}$$

$\Delta\tau_{i,j}$ pheromone amount to be dispersed on the edge $\overline{ij}$

Compute Objective Value of the tour generated (e.g. $f(x) = 15$)

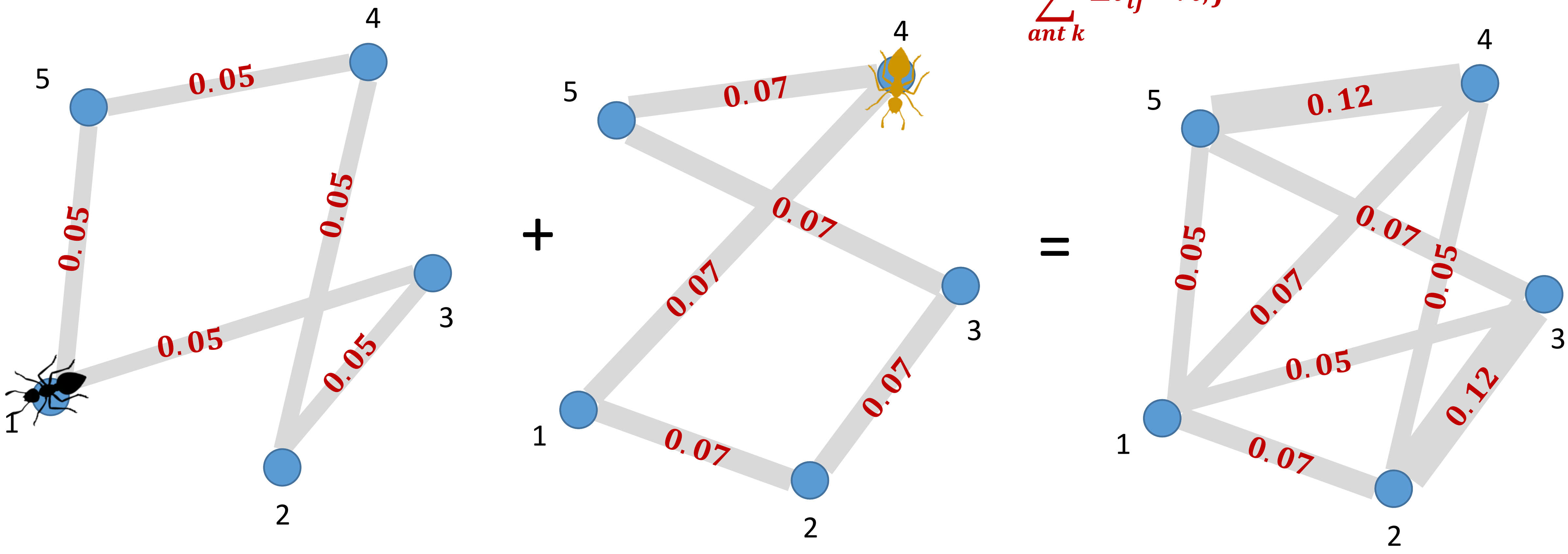
Compute the amount of pheromone to disperse in the path (e.g. $\Delta\tau_{ij} = \frac{1}{15} = 0.07$)

TSP Example

End of Iteration 1

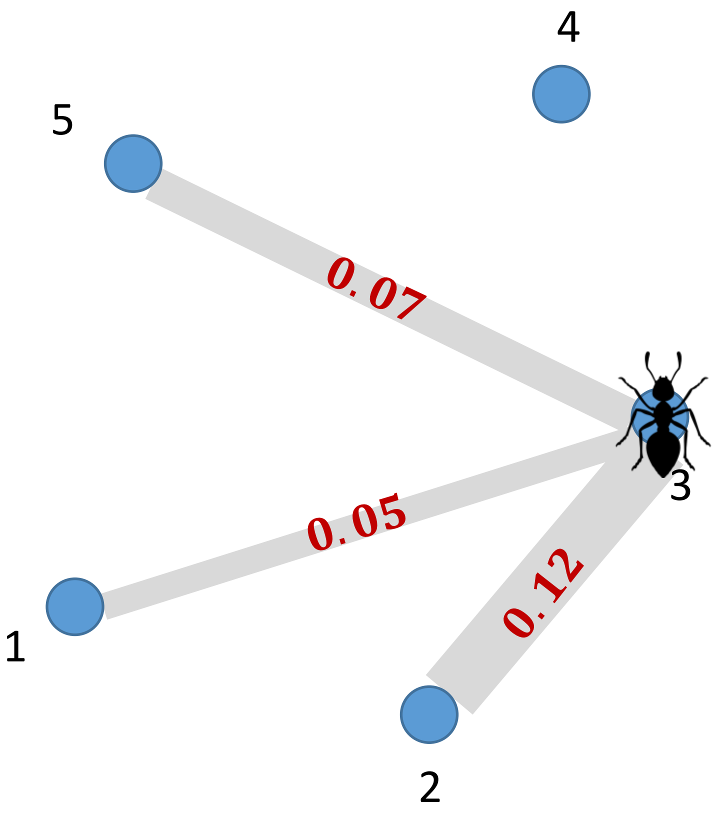
- Calculate the total pheromone amount in each edge

$$\sum_{ant\ k} \Delta\tau_{ij}^{k} \quad \forall i,j$$



TSP Example

Iteration 1 – Ant 1



Randomly select 1 city to locate 1 ant

Pheromones   Distance

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

$p_{i,j}$	probability of an ant to go to j if at location i	$\tau_{i,j}$	amount of pheromone on the edge connecting i and j
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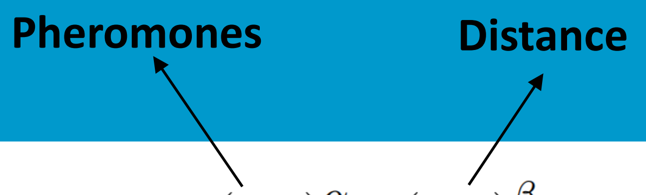
In the second iteration the decision depends on the amount of pheromone and the distance of the edges

Higher probability to select 2. Lower probability to select 4

TSP Example



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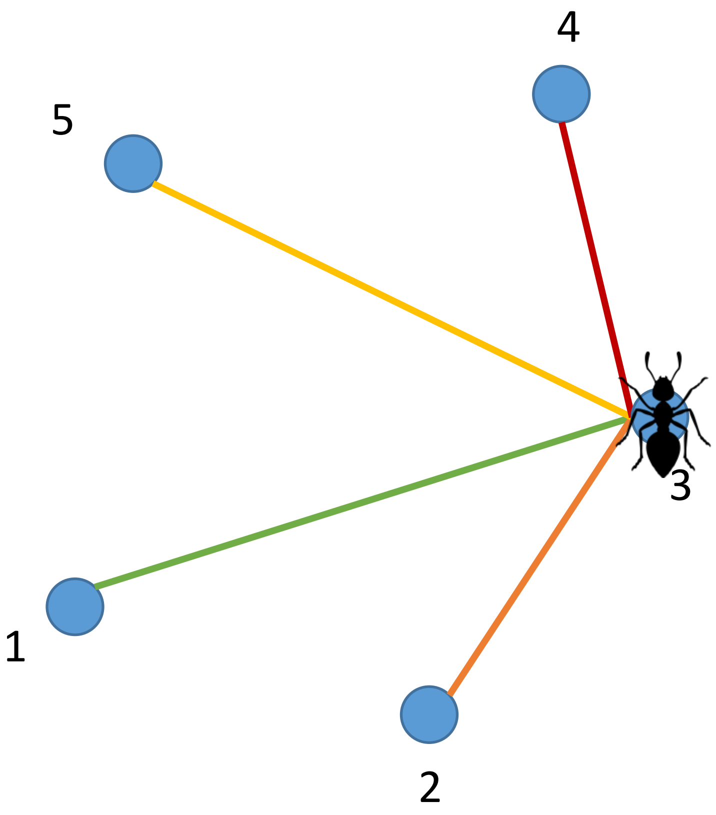
Iteration 1 – Ant 1



$p_{i,j}$ probability of an ant to go to j if at location i

α, β weight parameters

$\tau_{i,j}$ amount of pheromone on the edge connecting i and j

$\eta_{i,j}$ visibility of node j from i: inversely proportional to distance between j and i

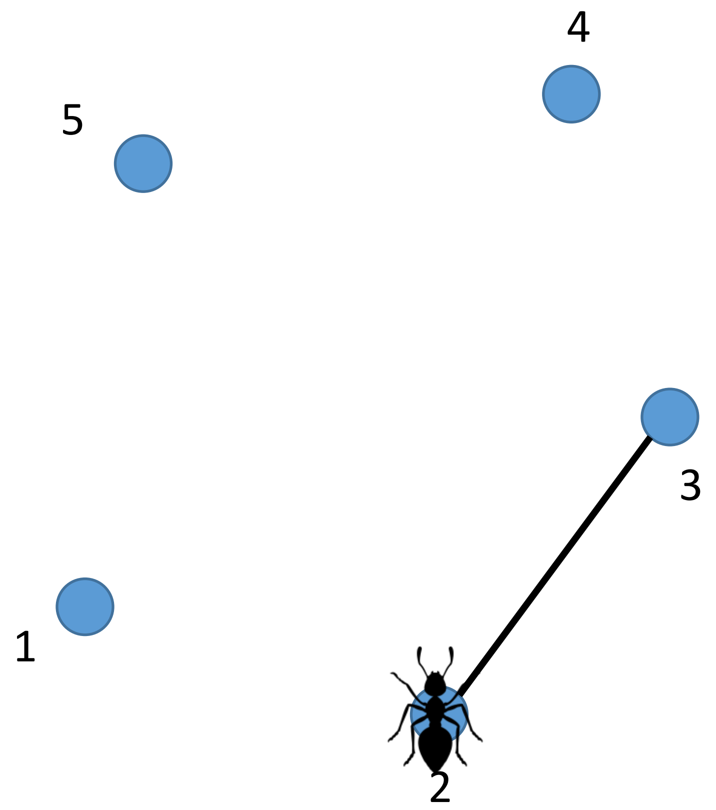
In the first iteration no pheromones are in the path
Ant selection of the next city to visit depends only on the distance
Randomized greedy approach (recall: lecture 6 - nearest neighbour heuristic)

Higher probability to select 4. Lower probability to select 1

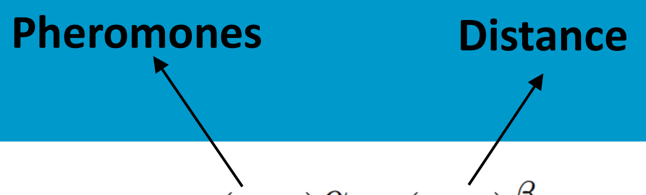
Randomly select 1 city to locate 1 ant

TSP Example

Iteration 1 – Ant 1



Randomly select 1 city to locate 1 ant



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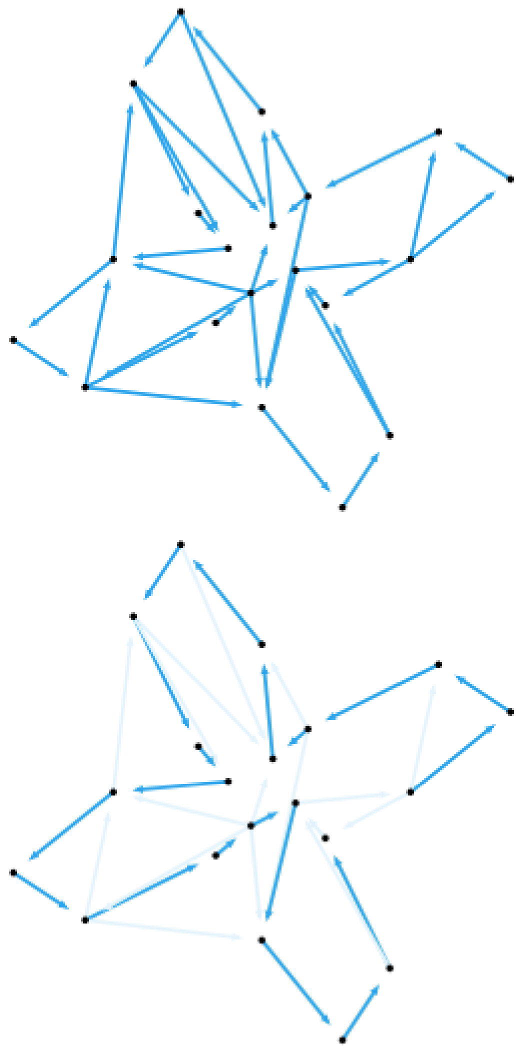
- $p_{i,j}$ — probability of an ant to go to j if at location i
- α, β — weight parameters
- $\tau_{i,j}$ — amount of pheromone on the edge connecting i and j
- $\eta_{i,j}$ — visibility of node j from i: inversely proportional to distance between j and i

REPEAT…

TSP Example



Solving the Job-shop Scheduling using ACO

Nuno Antunes Ribeiro

Assistant Professor

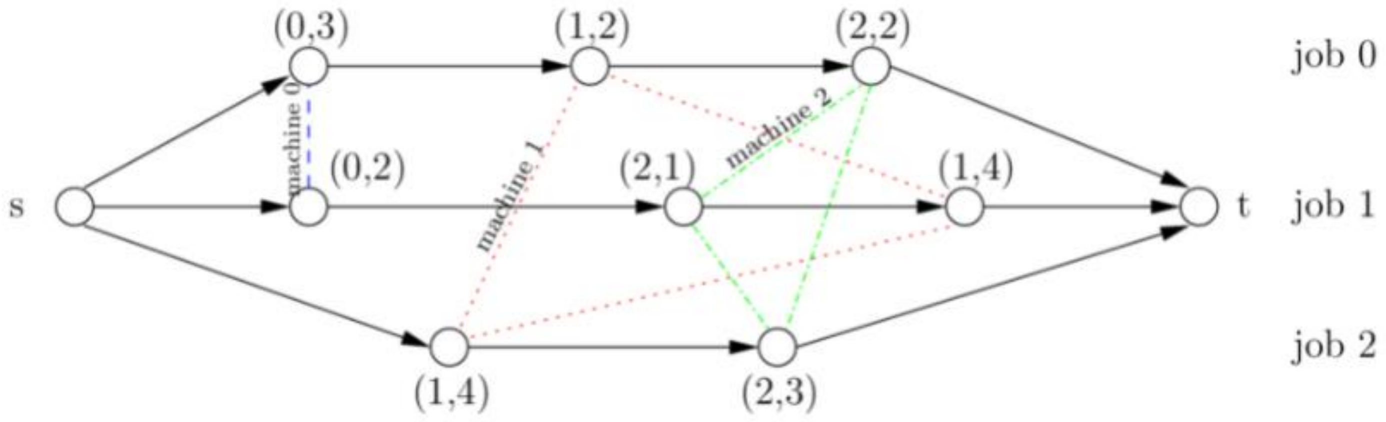
SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

Engineering Systems and Design

JSP Example

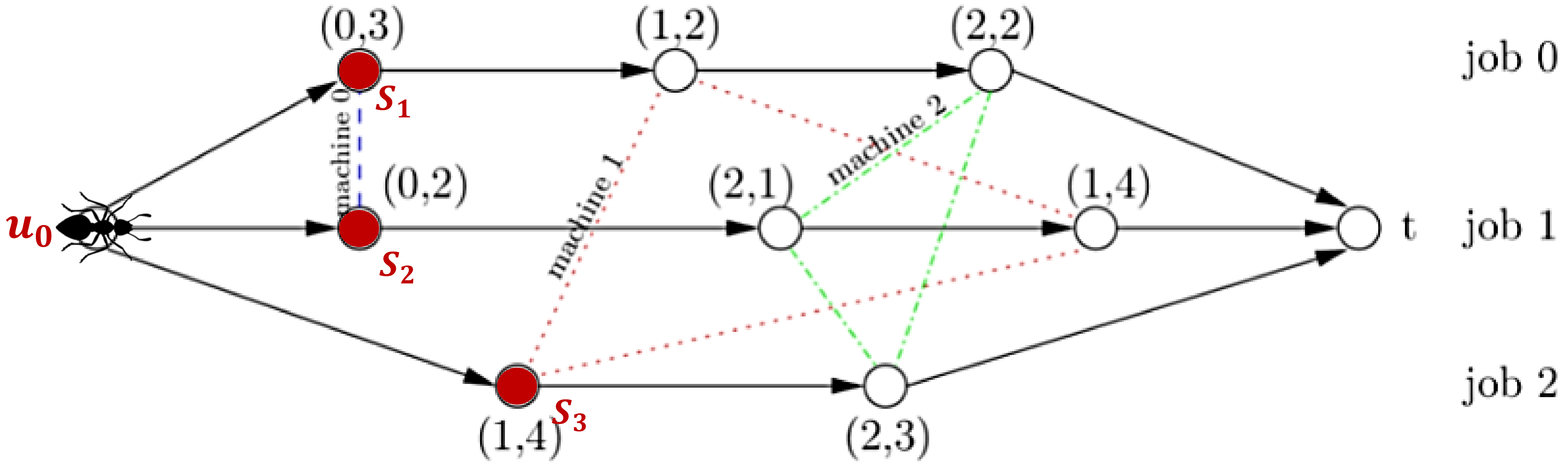
- A Gant chart can be represented as a disjunctive graph, which can be used to solve the Job-Shop Scheduling Problem using Ant Colony Optimization

Job	(Machine, Duration)	(Machine, Duration)	(Machine, Duration)
0	(0,3)	(1,2)	(2,2)
1	(0,2)	(2,1)	(1,4)
2	(1,4)	(2,3)	



JSP Example

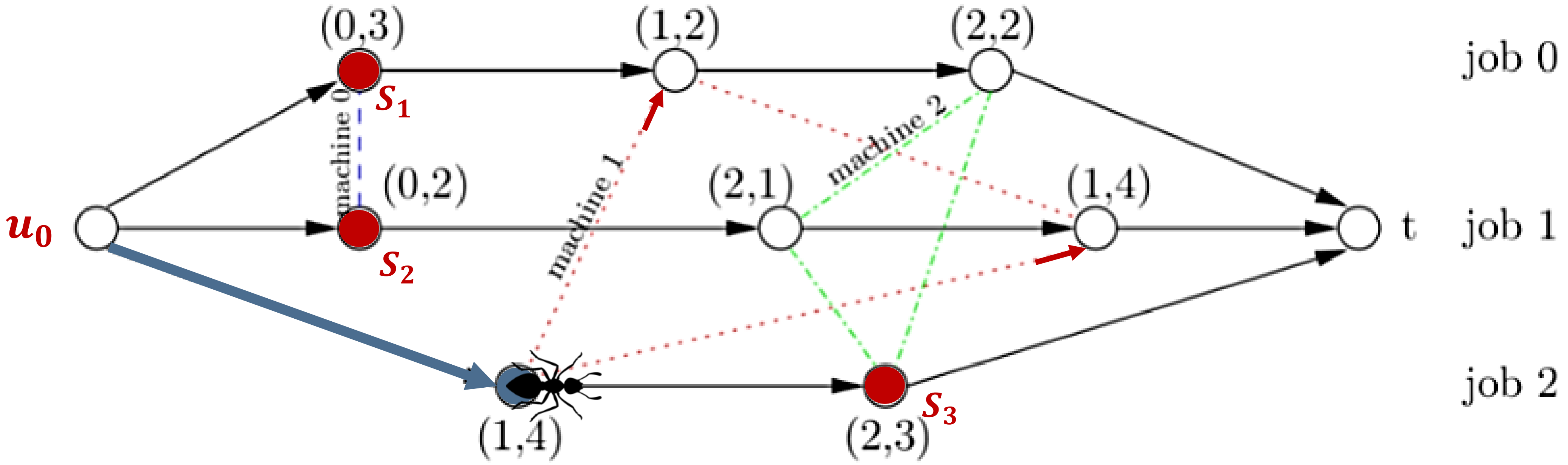
- All ants are initially in u_0 and are then left free to identify a permutation of the remaining nodes.
- In order to have a feasible permutation it is necessary to constrain the set of reachable nodes in any.
- Let G denote the set of all the nodes still to be visited and S the set of the nodes whose predecessors have already been visited.



JSP Example

Seq: J2T1;

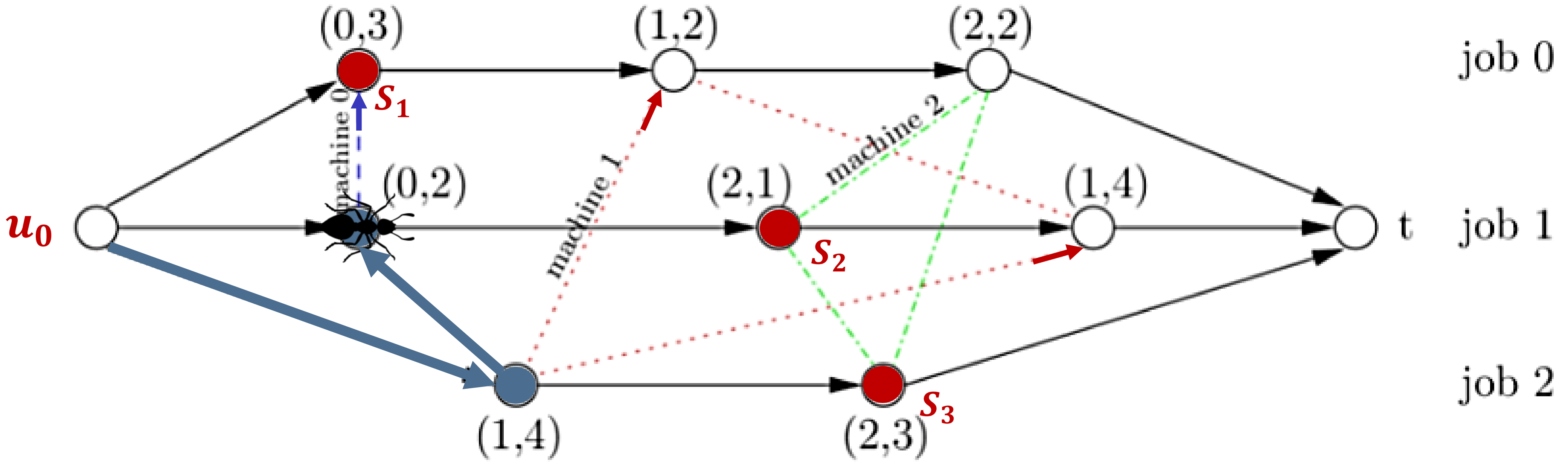
- All ants are initially in u_0 and are then left free to identify a permutation of the remaining nodes.
- In order to have a feasible permutation it is necessary to constrain the set of reachable nodes in any.
- Let G denote the set of all the nodes still to be visited and S the set of the nodes whose predecessors have already been visited.



JSP Example

Seq: J2T1; J1T0;

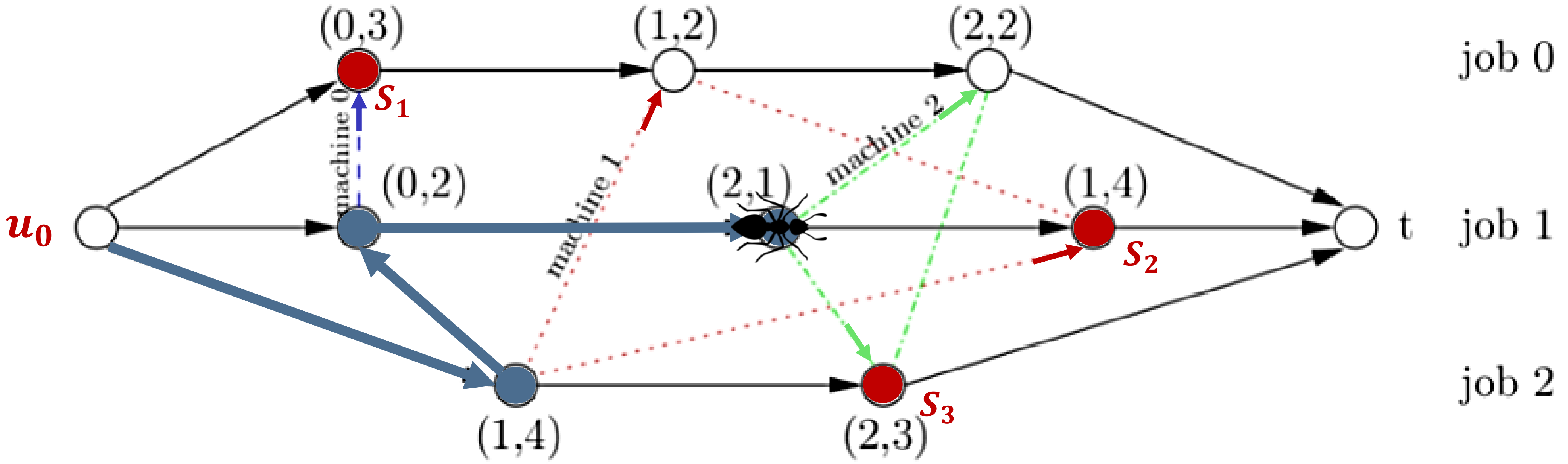
- All ants are initially in u_0 and are then left free to identify a permutation of the remaining nodes.
- In order to have a feasible permutation it is necessary to constrain the set of reachable nodes in any.
- Let G denote the set of all the nodes still to be visited and S the set of the nodes whose predecessors have already been visited.



JSP Example

Seq: J2T1; J1T0; J1T1;

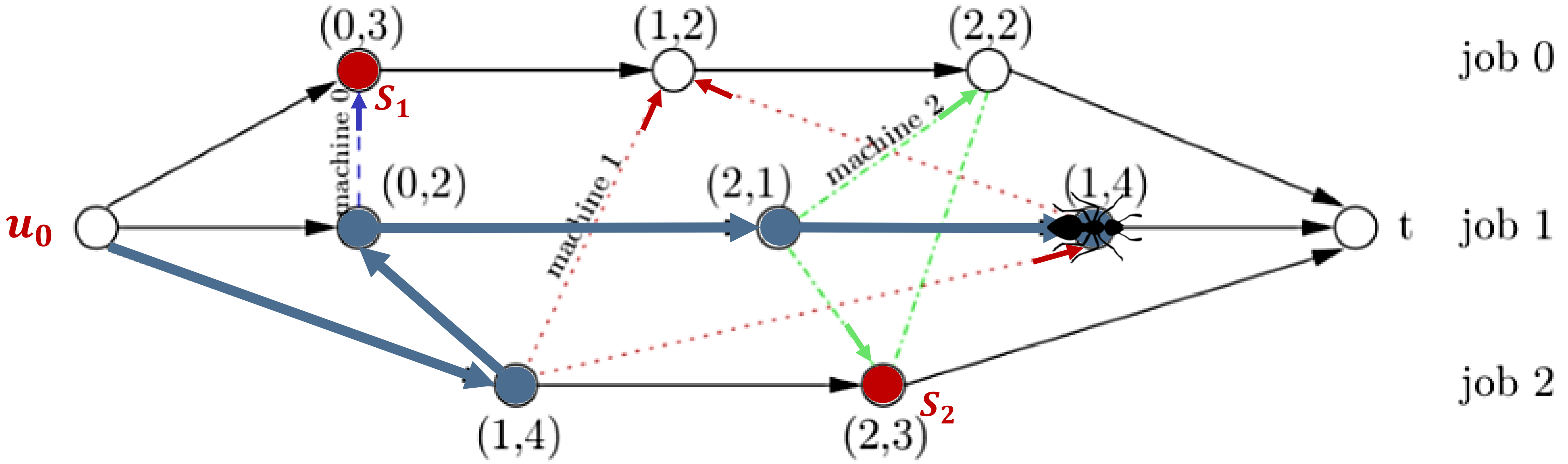
- All ants are initially in u_0 and are then left free to identify a permutation of the remaining nodes.
- In order to have a feasible permutation it is necessary to constrain the set of reachable nodes in any.
- Let G denote the set of all the nodes still to be visited and S the set of the nodes whose predecessors have already been visited.



JSP Example

Seq: J2T1; J1T0; J1T1; J1T2;

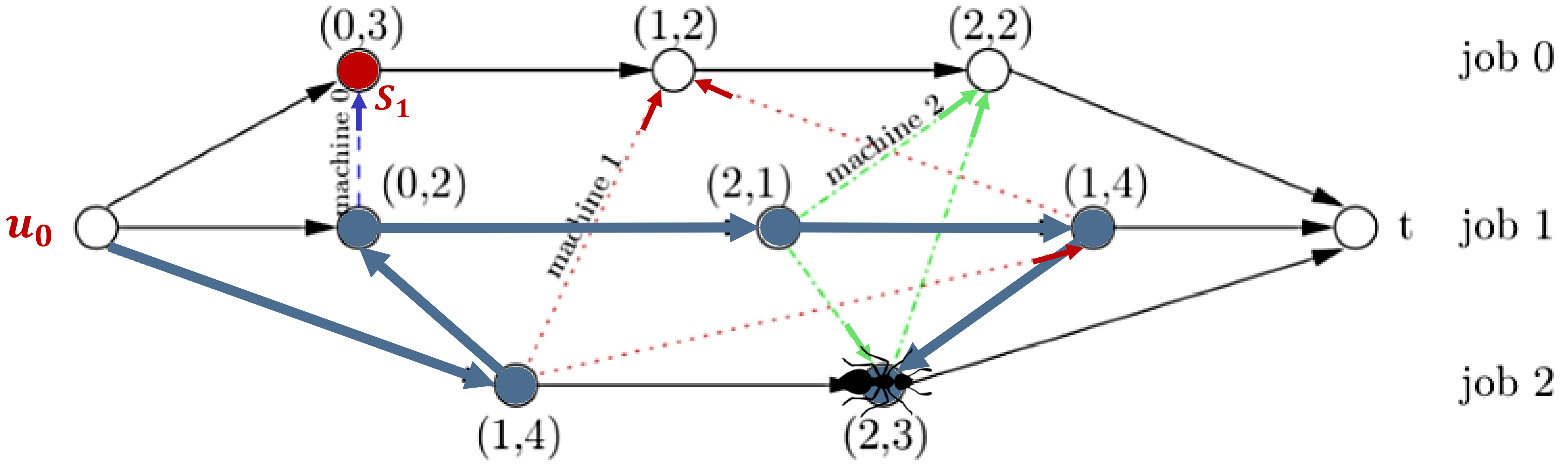
- All ants are initially in u_0 and are then left free to identify a permutation of the remaining nodes.
- In order to have a feasible permutation it is necessary to constrain the set of reachable nodes in any.
- Let G denote the set of all the nodes still to be visited and S the set of the nodes whose predecessors have already been visited.



JSP Example

Seq: J2T1; J1T0; J1T1; J1T2; J2T2;

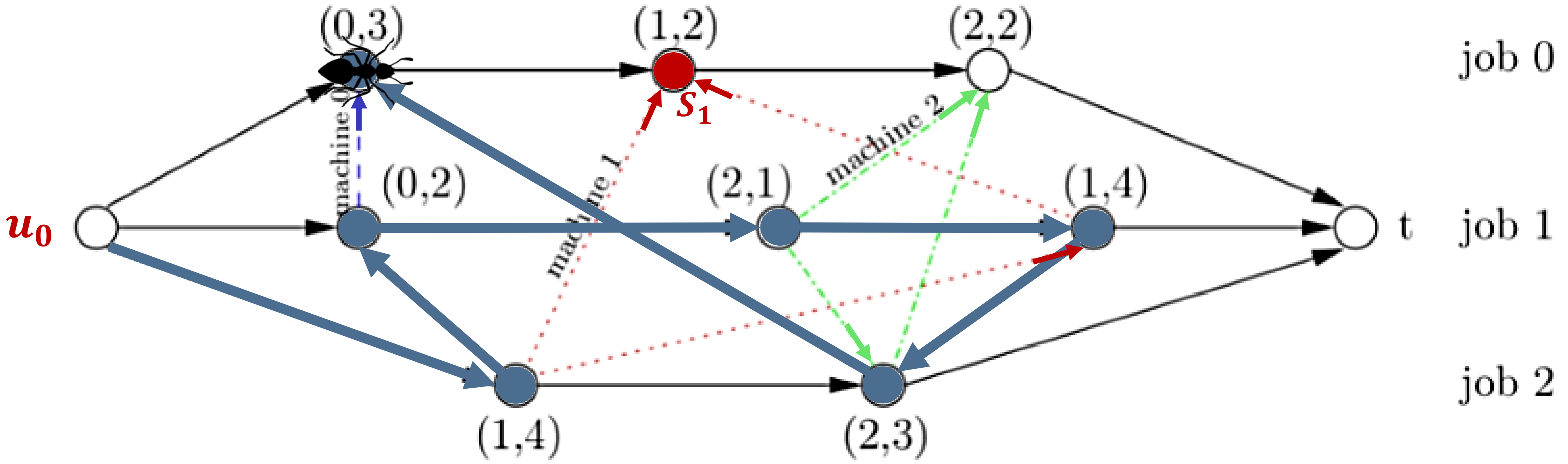
- All ants are initially in u_0 and are then left free to identify a permutation of the remaining nodes.
- In order to have a feasible permutation it is necessary to constrain the set of reachable nodes in any.
- Let G denote the set of all the nodes still to be visited and S the set of the nodes whose predecessors have already been visited.



JSP Example

Seq: J2T1; J1T0; J1T1; J1T2; J2T2; J0T0;

- All ants are initially in u_0 and are then left free to identify a permutation of the remaining nodes.
- In order to have a feasible permutation it is necessary to constrain the set of reachable nodes in any.
- Let G denote the set of all the nodes still to be visited and S the set of the nodes whose predecessors have already been visited.



JSP Example

Seq: J2T1; J1T0; J1T1; J1T2; J2T2; J0T0; J0T1;

- All ants are initially in u_0 and are then left free to identify a permutation of the remaining nodes.
- In order to have a feasible permutation it is necessary to constrain the set of reachable nodes in any.
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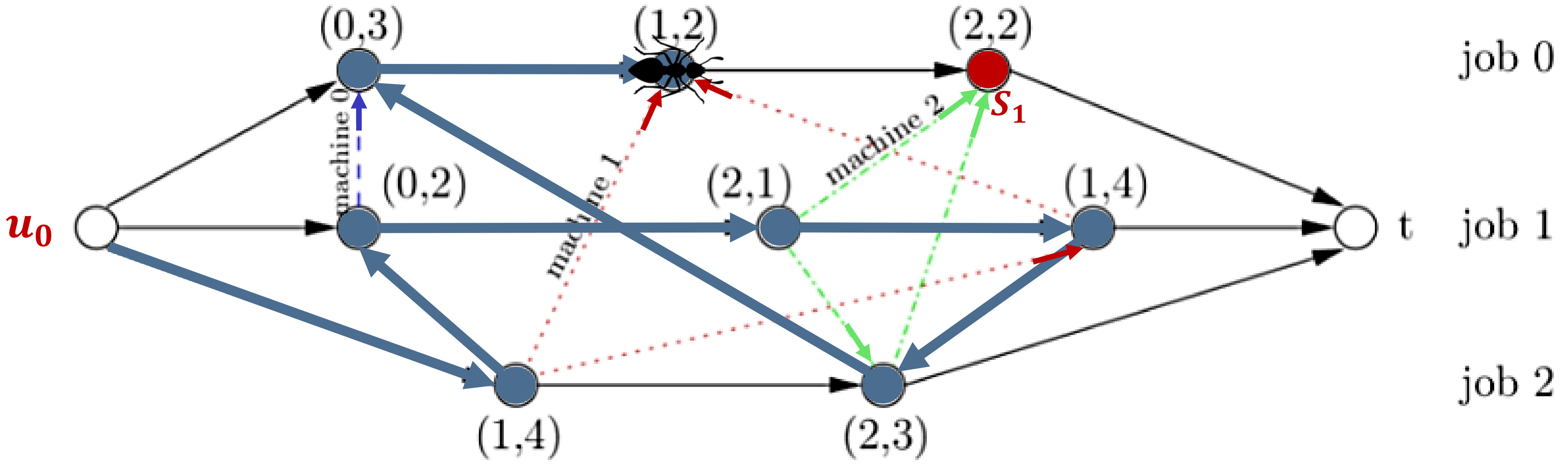

JSP Example

Seq: J2T1; J1T0; J1T1; J1T2; J2T2; J0T0; J0T1; J0T2

- All ants are initially in u_0 and are then left free to identify a permutation of the remaining nodes.
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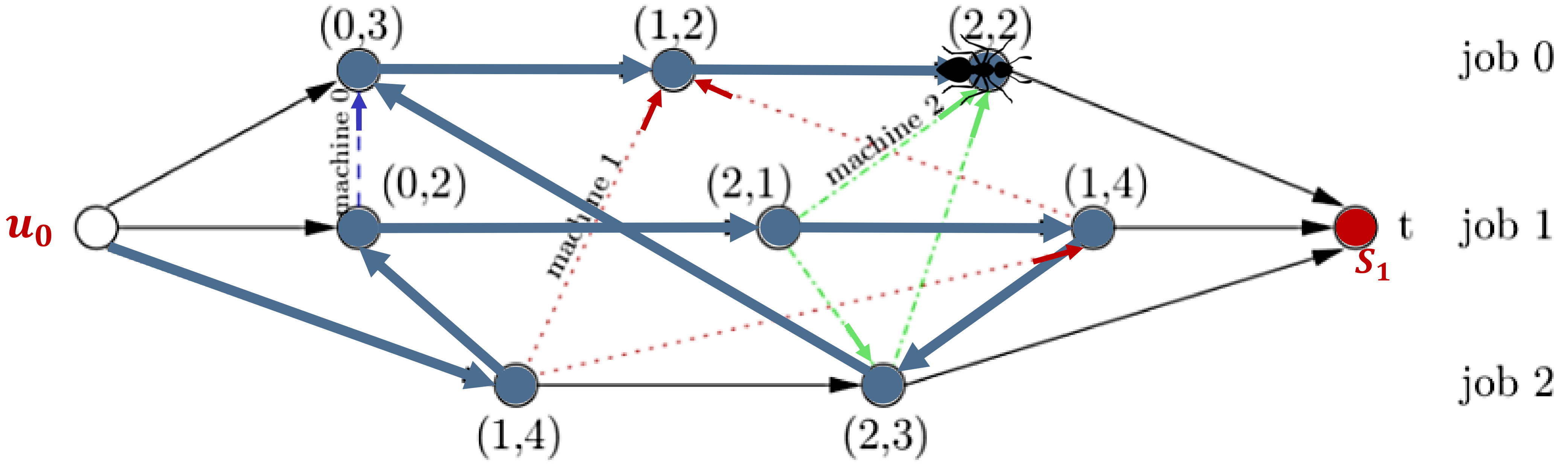

JSP Example

Seq: J2T1; J1T0; J1T1; J1T2; J2T2; J0T0; J0T1; J0T2

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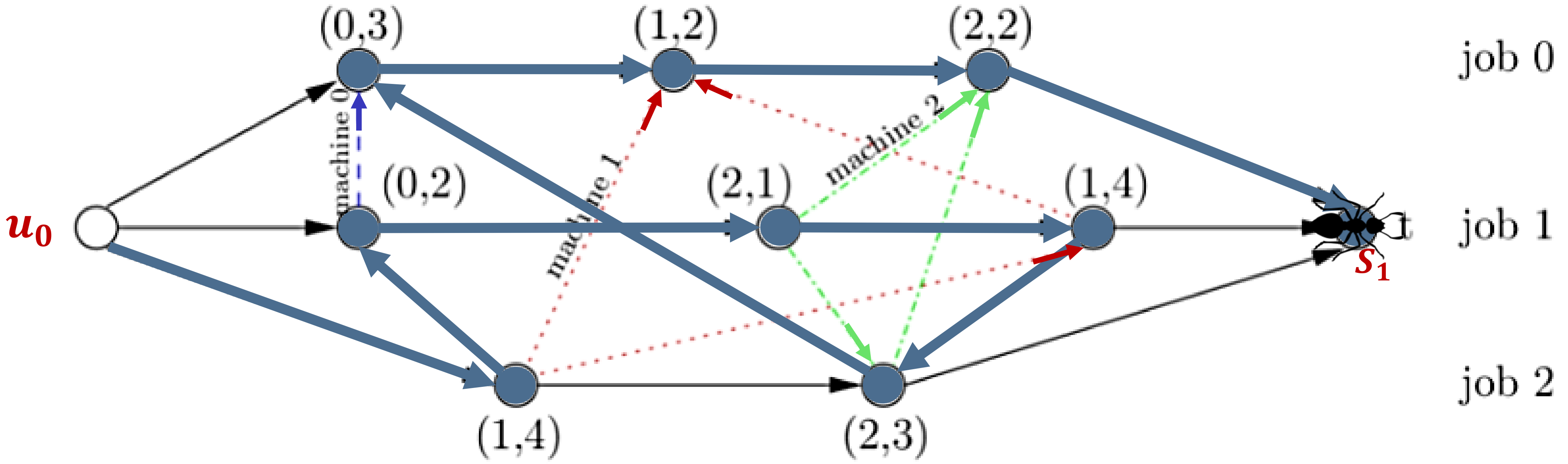
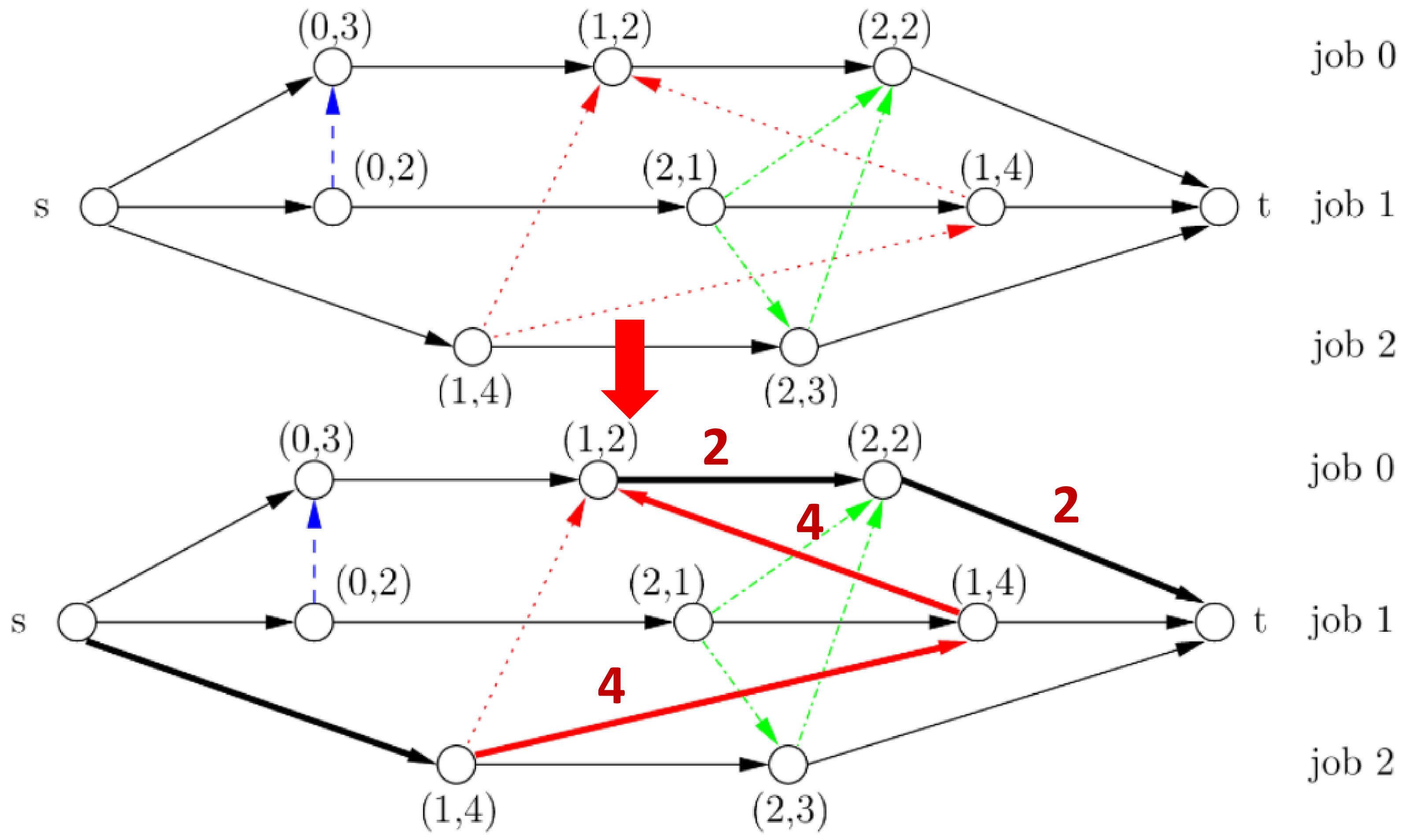


Recall: Critical Path (Slide 59 – Lec 4)

- Optimal critical path is given by the longest weighted path from *s* to *t*

Machine 0		Machine 1			Machine 2		
1	0	2	1	0	1	2	0



Critical Path Length =4+4+2+2=12

JSP Example

- How to compute η_{ij}?
  - Hypothesis 1: do not consider η_{ij} → $p_{i,j} = \frac{(\tau_{i,j})^\alpha}{\sum_{\forall k} (\tau_{i,k})^\alpha}$
  - Hypothesis 2: give more probability to operations with the longest (remaining) time

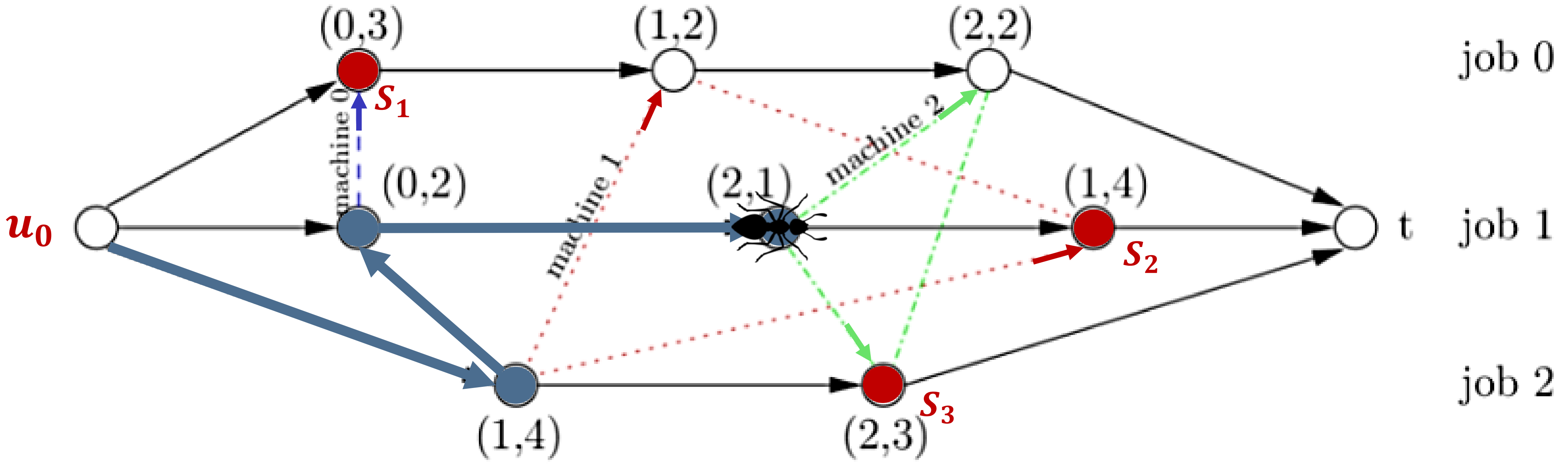
$$p_{i,j} = \frac{(\tau_{i,j})^\alpha * (\eta_{i,j})^\beta}{\sum_{\forall k} (\tau_{i,k})^\alpha * (\eta_{i,k})}$$

$p_{i,j}$	probability of an ant to go to j if at location i	$\tau_{i,j}$	amount of pheromone on the edge connecting i and j
α, β	weight parameters	$\eta_{i,j}$	visibility of node j from i: inversely proportional to distance between j and i

JSP Example

- Hypothesis 2: give more probability to operations with the longest (remaining) time

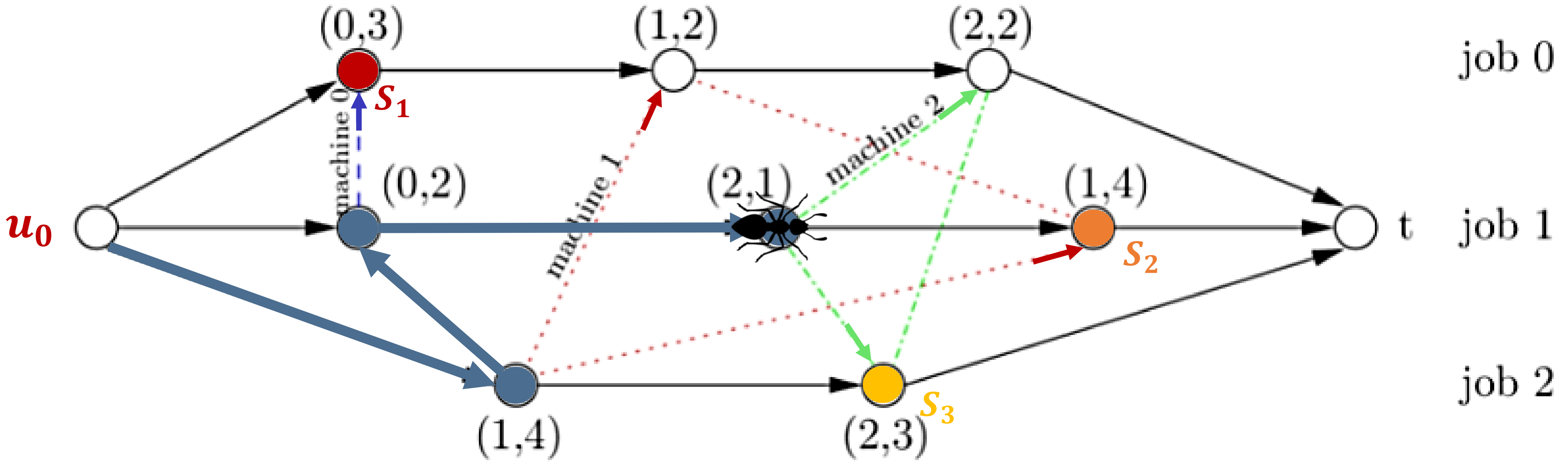
$\boldsymbol{\eta_{s_1} = 3 + 2 + 2 = 7}$ $\eta_{s_2} = 4$ $\eta_{s_3} = 3$



JSP Example

- Hypothesis 2: give more probability to operations with the longest (remaining) time

$\eta_{s_1} = 3 + 2 + 2 = 7$ $\eta_{s_2} = 4$ $\eta_{s_3} = 3$



Solving the Knapsack Problem using ACO

Nuno Antunes Ribeiro

Assistant Professor

Singapore University of Technology and Design | Engineering Systems and Design

Knapsack Problem Example

	w_i	f_i	f_i/w_i
Obj 1	2	5	2.5
Obj 2	3.75	7	1.87
Obj 3	2.5	3	1.2
Obj 4	3	5	1.67
Obj 5	1	4	4
Obj 6	1.5	8	5.33
cap	4		

Pheromones **Ratio (profit/weight)**

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

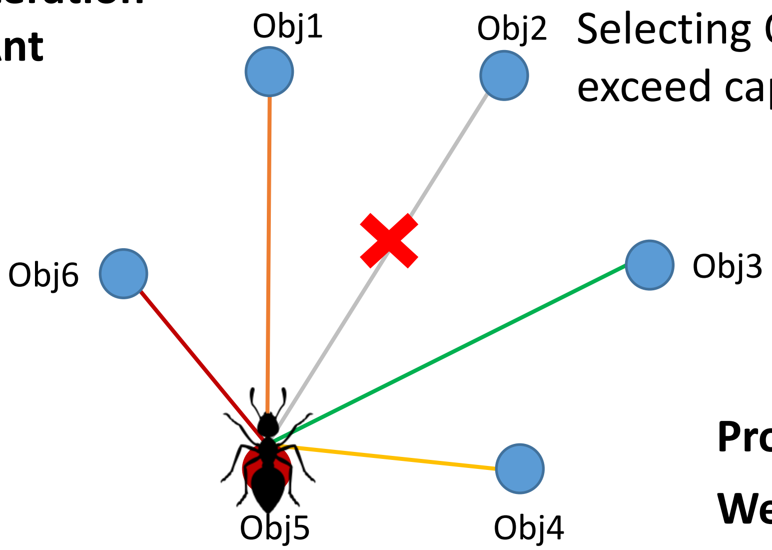
$p_{i,j}$ probability of an ant to go to j if at location i

$\tau_{i,j}$ amount of pheromone on the edge connecting i and j

α, β weight parameters

$\eta_{i,j}$ visibility of node j from i: inversely proportional to distance between j and i

1[st] Iteration
1[st] Ant



Selecting Obj 2 would exceed capacity

In the first iteration no pheromones are in the path
Ant selection depends only on the ratio (profit/weigh)

Profit=4

Weight=1

Higher probability to select 6. Lower probability to select 3
Object 2 cannot be selected

Knapsack Problem Example

	w_i	f_i	f_i/w_i
Obj 1	2	5	2.5
Obj 2	3.75	7	1.87
Obj 3	2.5	3	1.2
Obj 4	3	5	1.67
Obj 5	1	4	4
Obj 6	1.5	8	5.33
cap	4		

Pheromones **Ratio (profit/weight)**

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

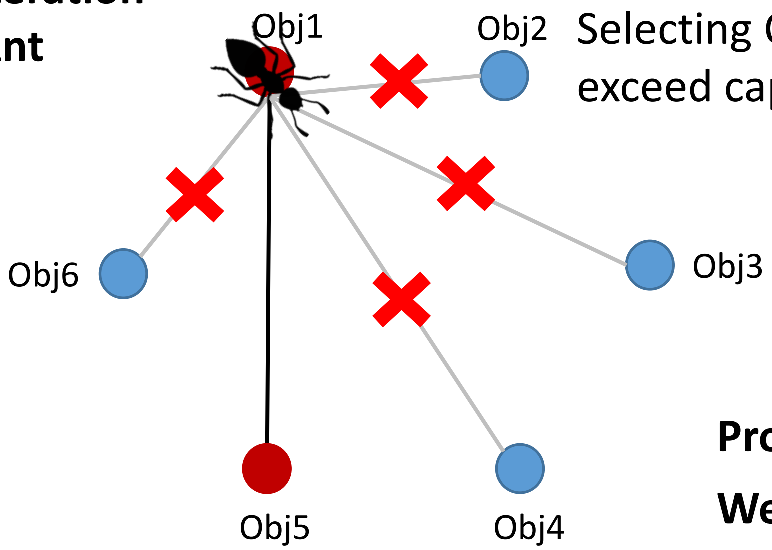
$p_{i,j}$ probability of an ant to go to j if at location i

α, β weight parameters

$\tau_{i,j}$ amount of pheromone on the edge connecting i and j

$\eta_{i,j}$ visibility of node j from i: inversely proportional to distance between j and i

1st Iteration
1st Ant



Selecting Obj 2 would exceed capacity

Profit=9

Weight=3

In the first iteration no pheromones are in the path
Ant selection depends only on the ratio (profit/weigh)

No more objects can be selected – end of the first iteration for ant 1

Knapsack Problem Example

	w_i	f_i	f_i/w_i
Obj 1	2	5	2.5
Obj 2	3.75	7	1.87
Obj 3	2.5	3	1.2
Obj 4	3	5	1.67
Obj 5	1	4	4
Obj 6	1.5	8	5.33
cap	4		

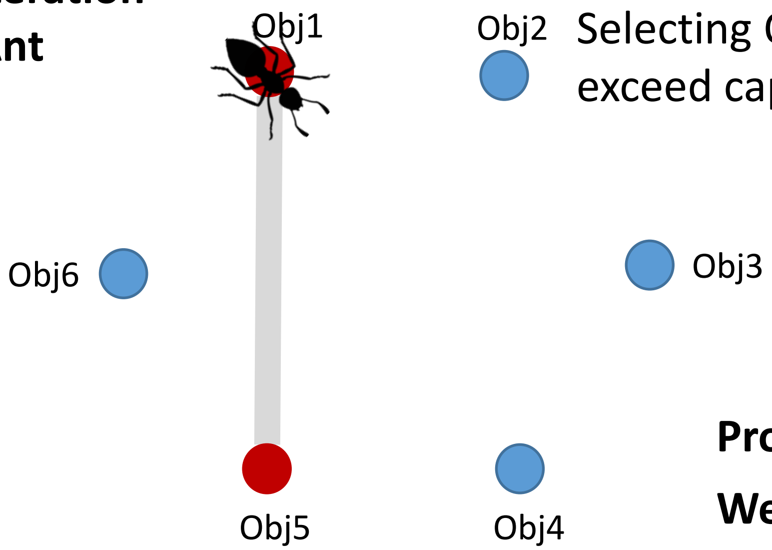
$$\Delta\tau_{i,j} = \begin{cases} 1 - \frac{1}{f(x)} & \text{if tour } x \text{ contains edge } \overline{ij} \\ 0 & \text{otherwise} \end{cases}$$

$\Delta\tau_{i,j}$ pheromone amount to be dispersed on the edge $\overline{ij}$

Compute Objective Value of the objects selected (e.g. $f(x) = 9$)

Compute the amount of pheromone to disperse in the path (e.g. $\Delta\tau_{ij} = 1 - \frac{1}{9} = 0.88$)

1st Iteration
1st Ant



Selecting Obj 2 would exceed capacity

Profit=9

Weight=3

Knapsack Problem Example

	w_i	f_i	f_i/w_i
Obj 1	2	5	2.5
Obj 2	3.75	7	1.87
Obj 3	2.5	3	1.2
Obj 4	3	5	1.67
Obj 5	1	4	4
Obj 6	1.5	8	5.33
cap	4		

Pheromones **Ratio (profit/weight)**

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

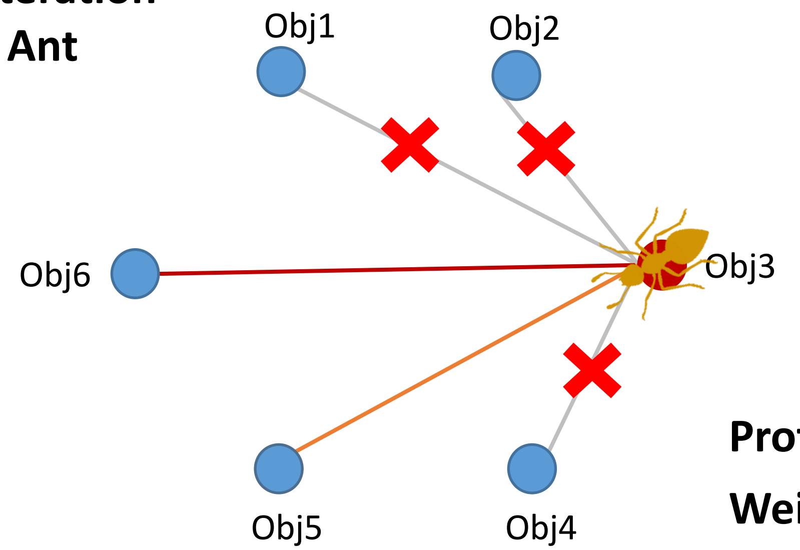
$p_{i,j}$ probability of an ant to go to j if at location i

$\tau_{i,j}$ amount of pheromone on the edge connecting i and j

α, β weight parameters

$\eta_{i,j}$ visibility of node j from i: inversely proportional to distance between j and i

1st Iteration
2nd Ant



Profit=3

Weight=2.5

In the first iteration no pheromones are in the path
Ant selection depends only on the ratio (profit/weigh)

Higher probability to select 6. Lower probability to select 5
Object 2 cannot be selected

Knapsack Problem Example

	w_i	f_i	f_i/w_i
Obj 1	2	5	2.5
Obj 2	3.75	7	1.87
Obj 3	2.5	3	1.2
Obj 4	3	5	1.67
Obj 5	1	4	4
Obj 6	1.5	8	5.33
cap	4		

Pheromones **Ratio (profit/weight)**

$$p_{i,j} = \frac{(\tau_{i,j})^{\alpha} * (\eta_{i,j})^{\beta}}{\sum_{\forall k} (\tau_{i,k})^{\alpha} * (\eta_{i,k})}$$

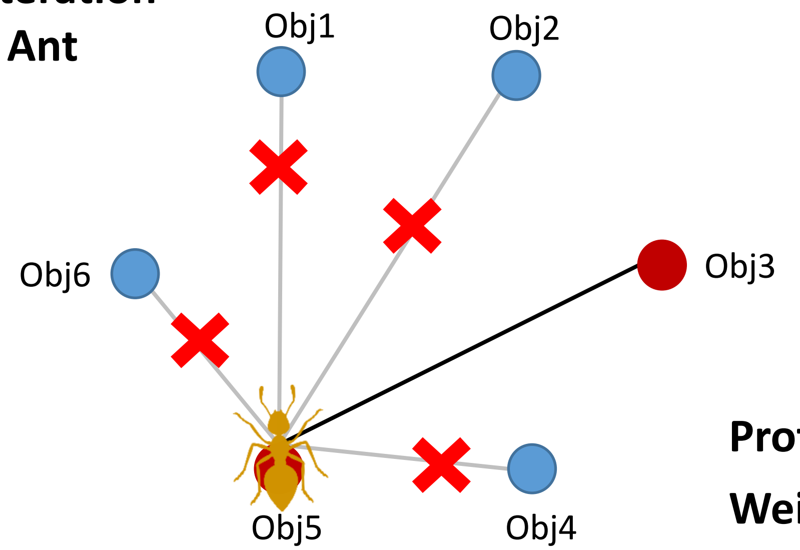
$p_{i,j}$ probability of an ant to go to j if at location i

$\tau_{i,j}$ amount of pheromone on the edge connecting i and j

α, β weight parameters

$\eta_{i,j}$ visibility of node j from i: inversely proportional to distance between j and i

1st Iteration
2nd Ant



Profit=7

Weight=3.5

In the first iteration no pheromones are in the path
Ant selection depends only on the ratio (profit/weigh)

No more objects can be selected – end of the first iteration for ant 2

Knapsack Problem Example

	w_i	f_i	f_i/w_i
Obj 1	2	5	2.5
Obj 2	3.75	7	1.87
Obj 3	2.5	3	1.2
Obj 4	3	5	1.67
Obj 5	1	4	4
Obj 6	1.5	8	5.33
cap	4		

$$\Delta\tau_{i,j} = \begin{cases} 1 - \frac{1}{f(x)} & \text{if tour } x \text{ contains edge } \overline{ij} \\ 0 & \text{otherwise} \end{cases}$$

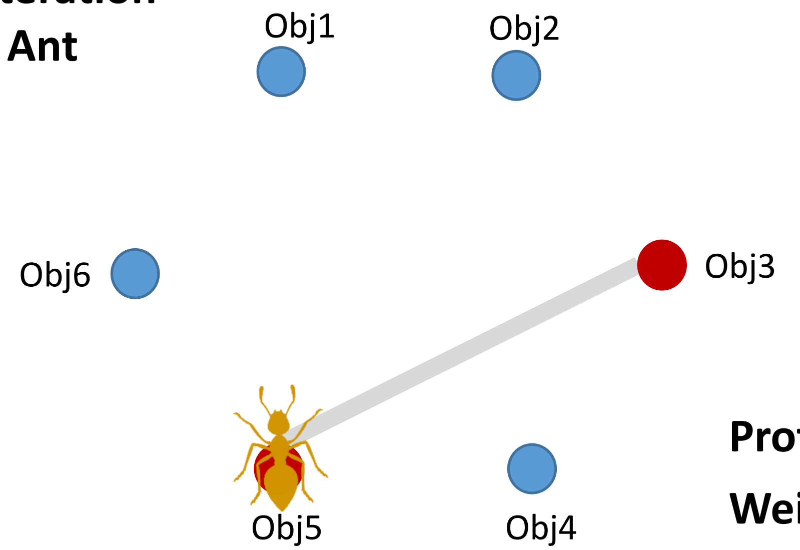
$\Delta\tau_{i,j}$ pheromone amount to be dispersed on the edge $\overline{ij}$

Compute Objective Value of the objects selected (e.g. $f(x) = 7$)

Compute the amount of pheromone to disperse in the path (e.g. $\Delta\tau_{ij} = \frac{1}{7} = 0.85$)

1st Iteration

2nd Ant



Profit=7

Weight=3.5

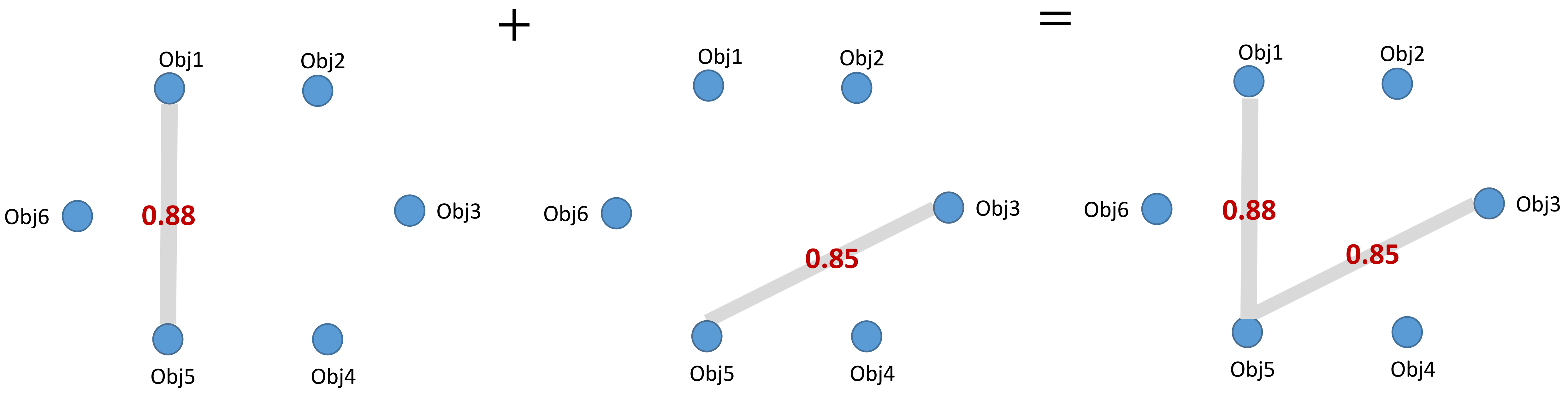
TSP Example

Ant Colony is essentially a randomized greedy algorithm with memory – the ant's moves are the greedy steps, the amount of pheromone is the memory

End of Iteration 1

- Calculate the total pheromone amount in each edge

$$\sum_{ant\ k} \Delta\tau_{ij}^{k} \quad \forall i, j$$



Repeat, now with pheromones