# Solution Encoding and Search Operators


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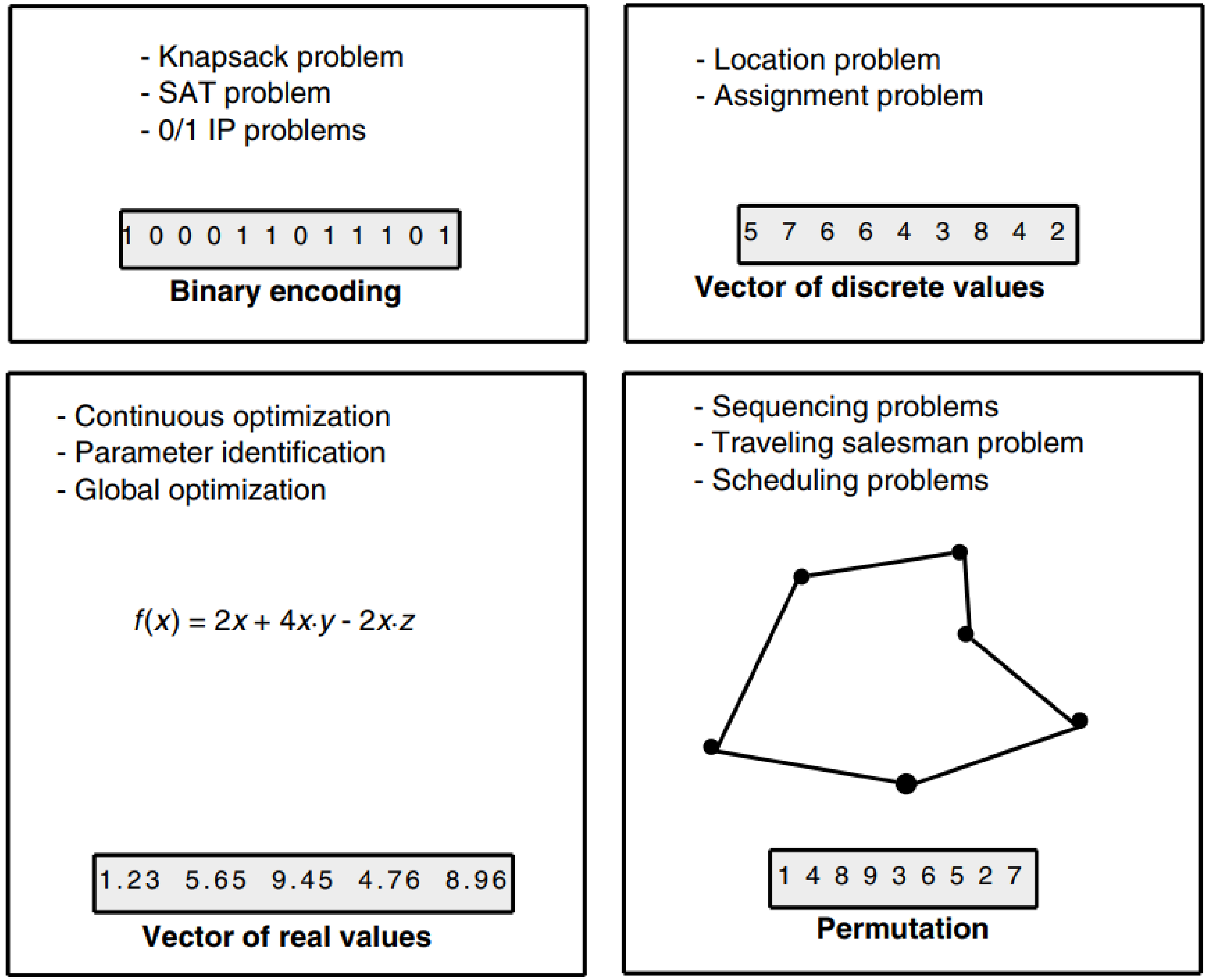
# Solution Encoding

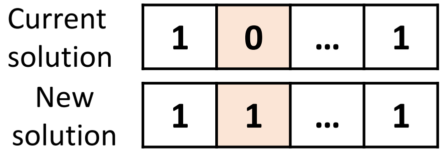
- Designing any iterative metaheuristic needs an **encoding** of a solution
- The encoding plays a major role in the efficiency and effectiveness of a metaheuristic procedure and constitutes an essential step in designing **any** metaheuristic.
- Many straightforward encodings may be applied for some traditional families of optimization problems. Those representations may be combined or underlying new representations.



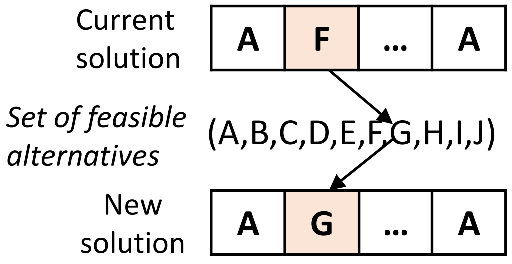
# Search Operator

- The efficiency of a solution encoding is also related to the **search operator**.
- When defining a solution encoding, one has to bear in mind how the solution will be **perturbed**.
- **Default search operators** are often considered – however more sophisticated search operators may be considered, especially when solving problems with large solution spaces and a significant number of constraints.

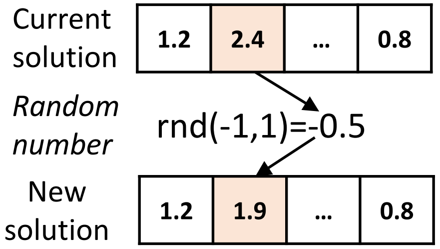
**Binary encoding –** flip n bits of the solution (typically 1 or 2 bits)



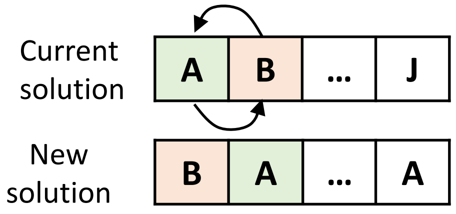

**Discrete encoding –** update n bits of the solution by randomly generating a new value (typically 1 or 2 bits)




**Real encoding –** update n bits of the solution by randomly generating a new value within a certain range (typically 2 elements)




**Permutation encoding –** swap the location of n elements (typically 2 elements)

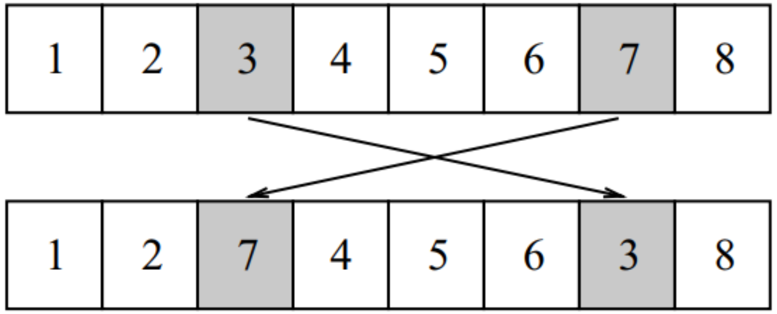


# Search Operators in Permutation Problems

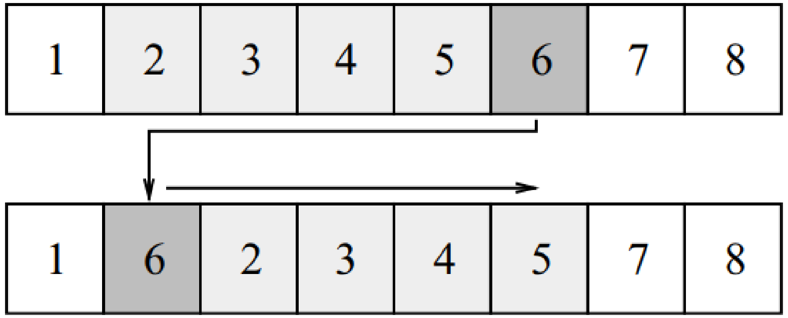
- Many **sequencing, scheduling, planning and routing problems** are considered as permutation problems
- There are two main types of permutation problems:
  - **Priority Problems** (e.g. **scheduling**) - In these problems permutations represent a priority queue and the position in the solution is important
  - **Adjacency Problems** (e.g. **TSP**) - In these problems permutations represent an adjacency list - e.g. city "A" may be the first in the list, the second, or the nth element in the list; the solution is the same provided that all elements of the list are adjacent to the same pair of elements
- **The efficiency of a neighbourhood is related not only to the representation but also the type of problems to solve and corresponding search operators**

# Search Operators in Permutation Problems

- Swap Operator

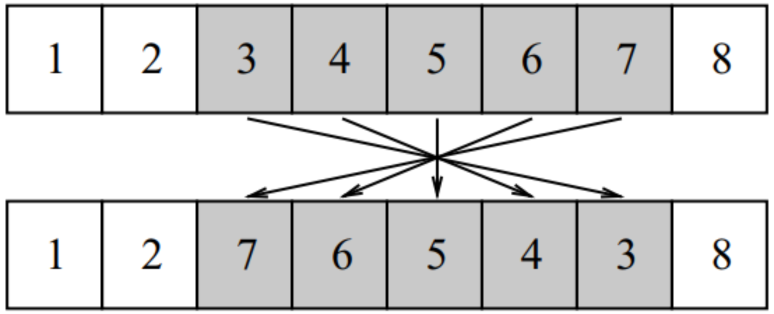



- Insertion Operator



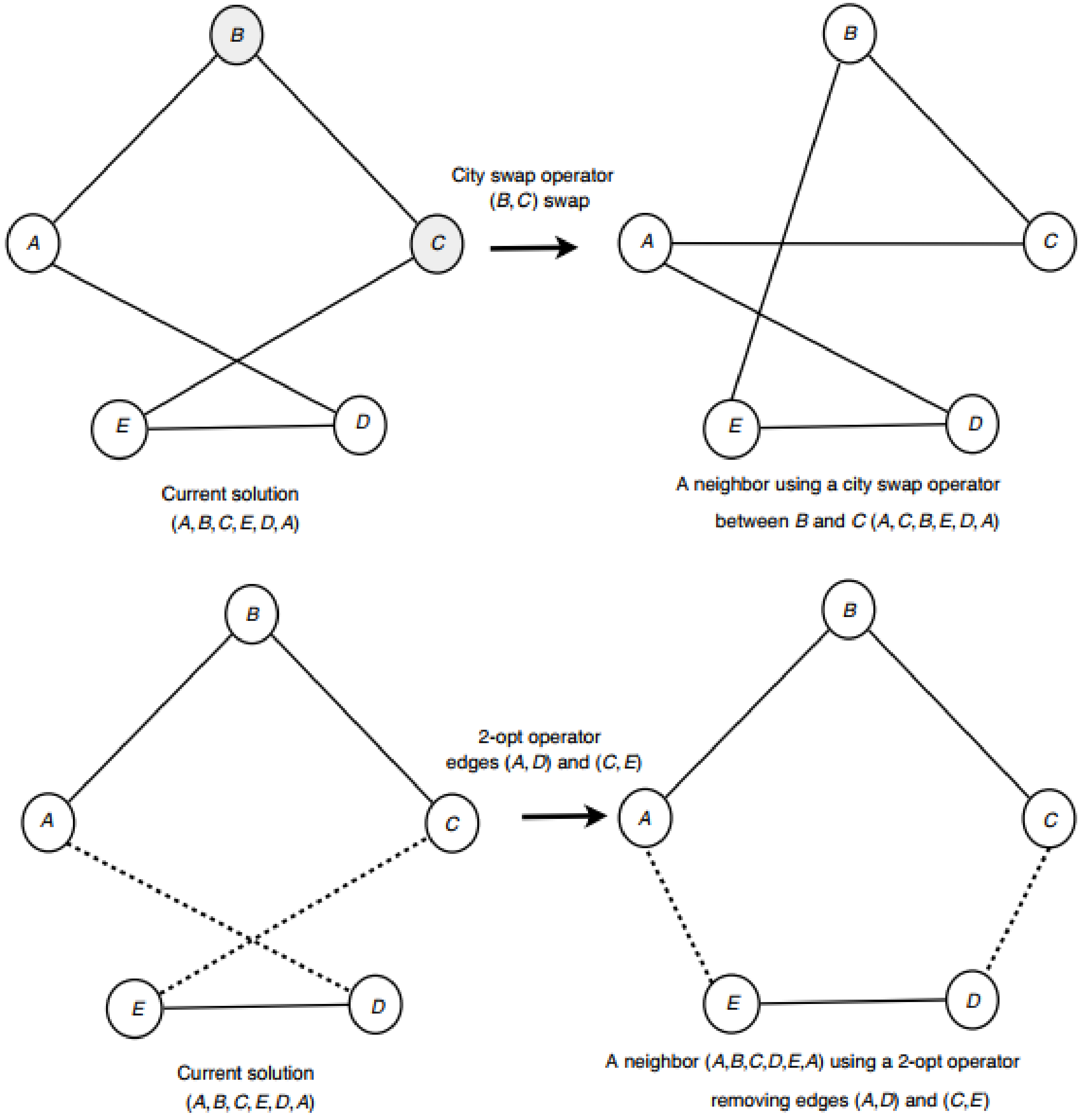

**Used in both: Priority and Adjacency Problems**

- Inversion Operator



# Search Operators in Adjacency Problems

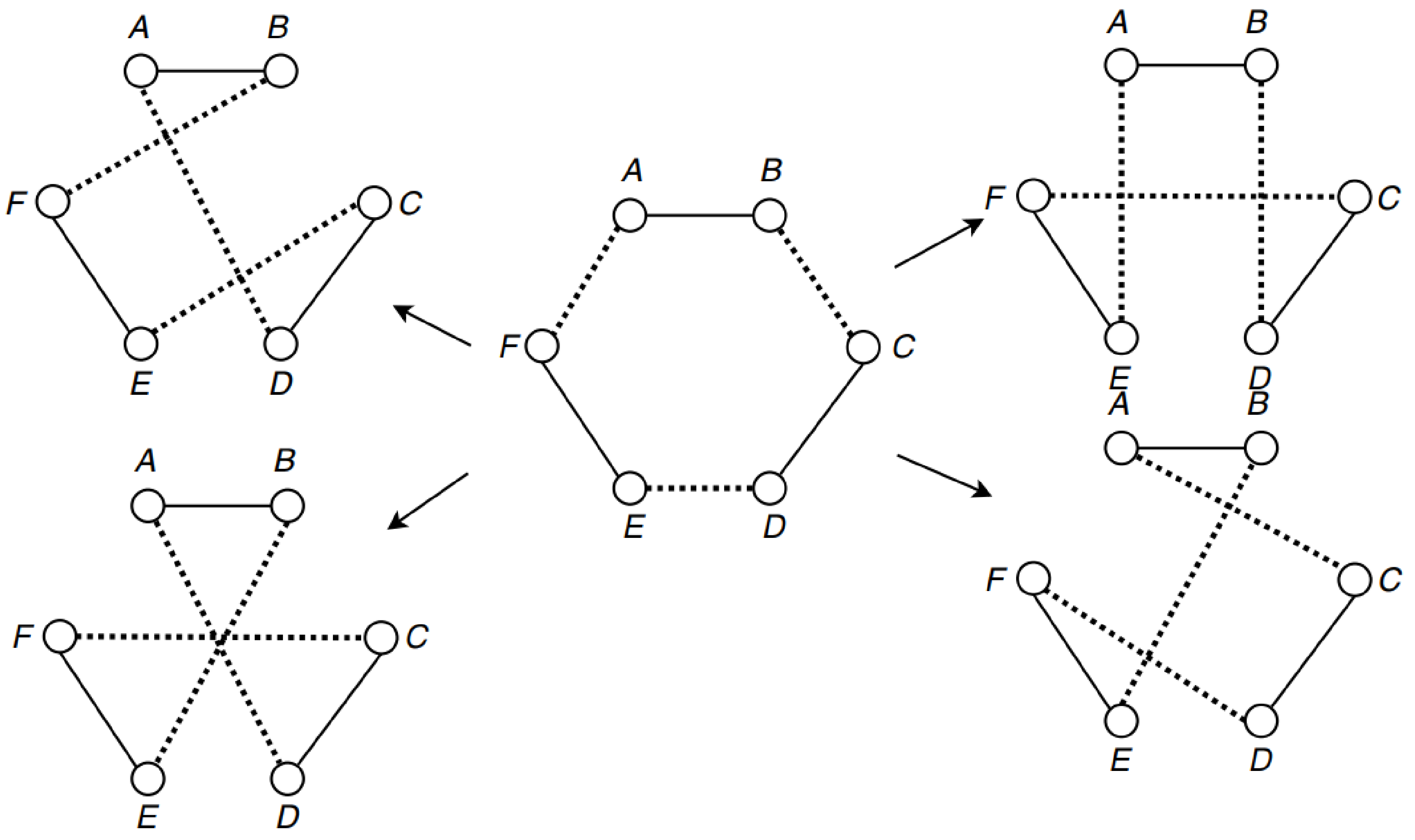
- 2-Opt




**Only used in Adjacency Problems**

# Search Operators in Adjacency Problems

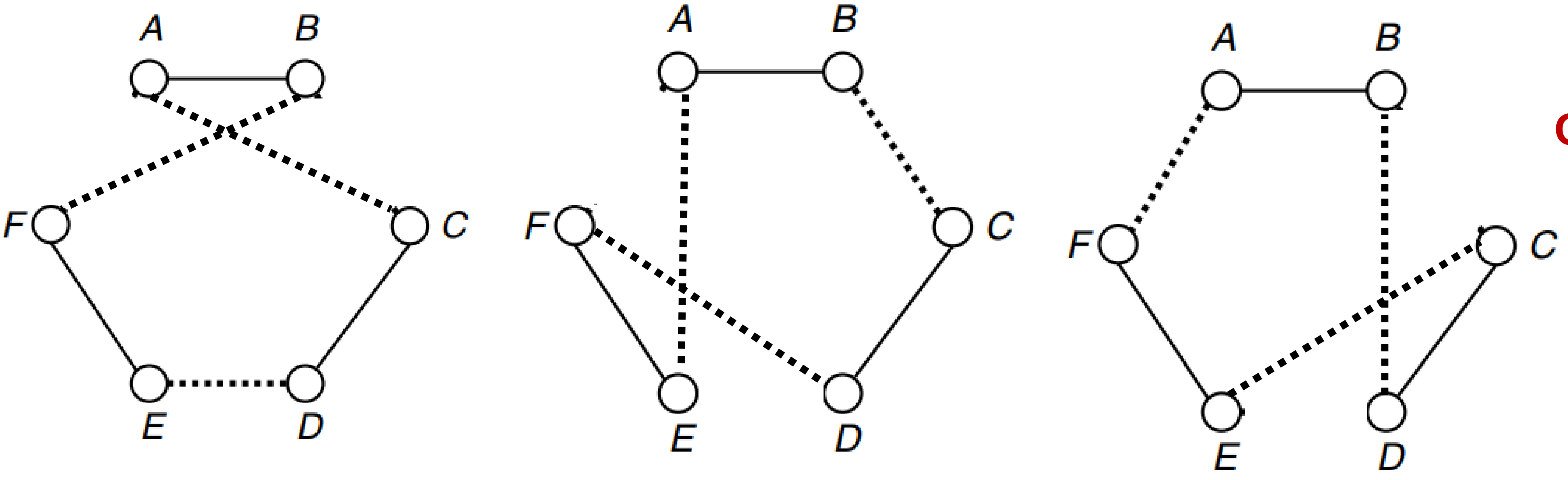
- 3-Opt




**Only used in Adjacency Problems**

# Search Operators in Adjacency Problems

- 3-Opt (2Opt Solutions)




**Only used in Adjacency Problems**

3-Opt Neighbourhood – 7 Solutions

# Search Operators in Adjacency Problems

- 3-Opt (Generalization)
  - Split the tour into 3 random parts (A – B – C)
  - 3-Opt Solutions:
    - A – inv(B) – inv(C)
    - A – C – B
    - A – C – inv(B)
    - A – inv(C) – B
  - 2-Opt Solutions:
    - A – inv(B) – C
    - A – B – inv(C)
    - A – inv(C) – inv(B)

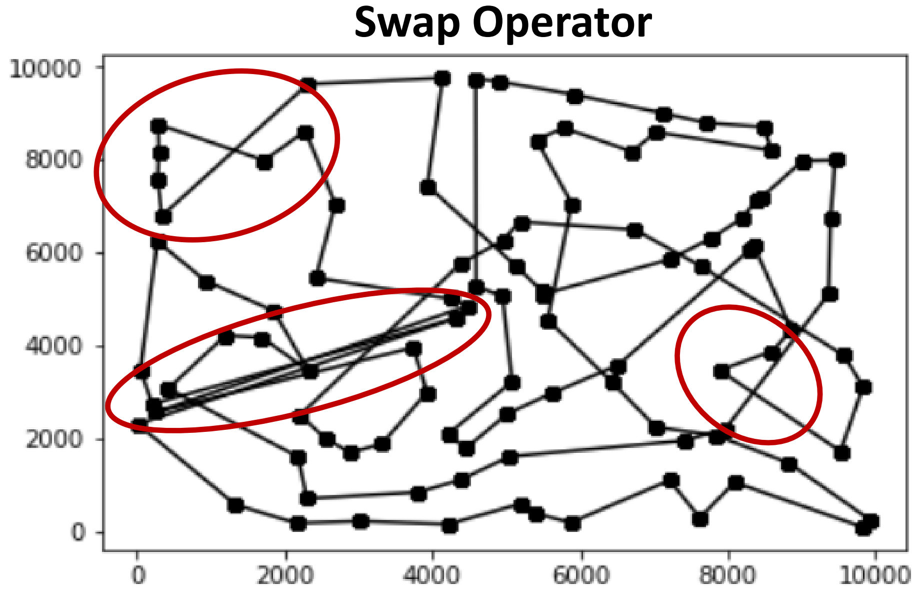
*inv stands for inversion

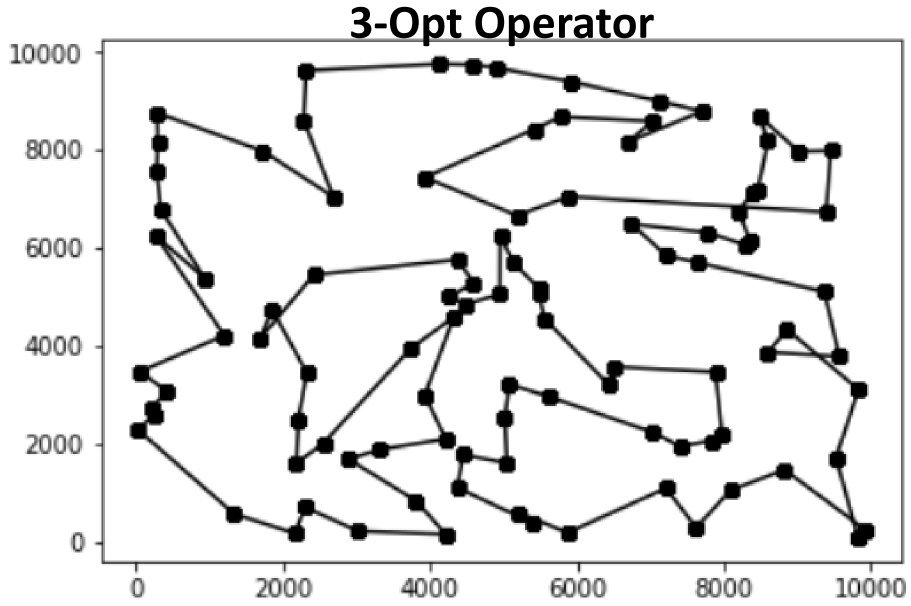
| | | A | | | B | | | C | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

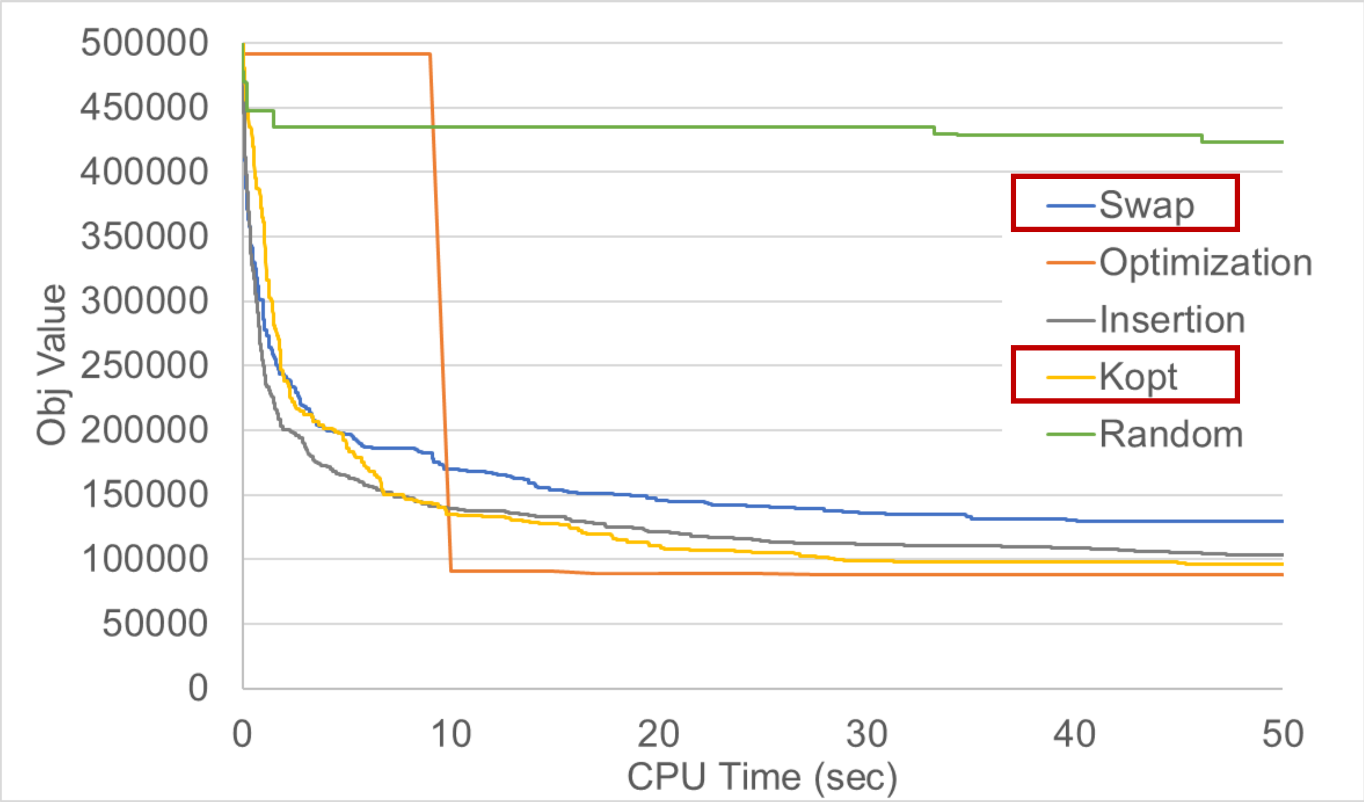
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 7 | 6 | 10 | 9 | 8 |
| 1 | 2 | 3 | 4 | 5 | 8 | 9 | 10 | 6 | 7 |
| 1 | 2 | 3 | 4 | 5 | 8 | 9 | 10 | 7 | 6 |
| 1 | 2 | 3 | 4 | 5 | 10 | 9 | 8 | 6 | 7 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 7 | 6 | 8 | 9 | 10 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 10 | 9 | 8 |
| 1 | 2 | 3 | 4 | 5 | 10 | 9 | 8 | 7 | 6 |

# Swap Operator vs 3-Opt Operator

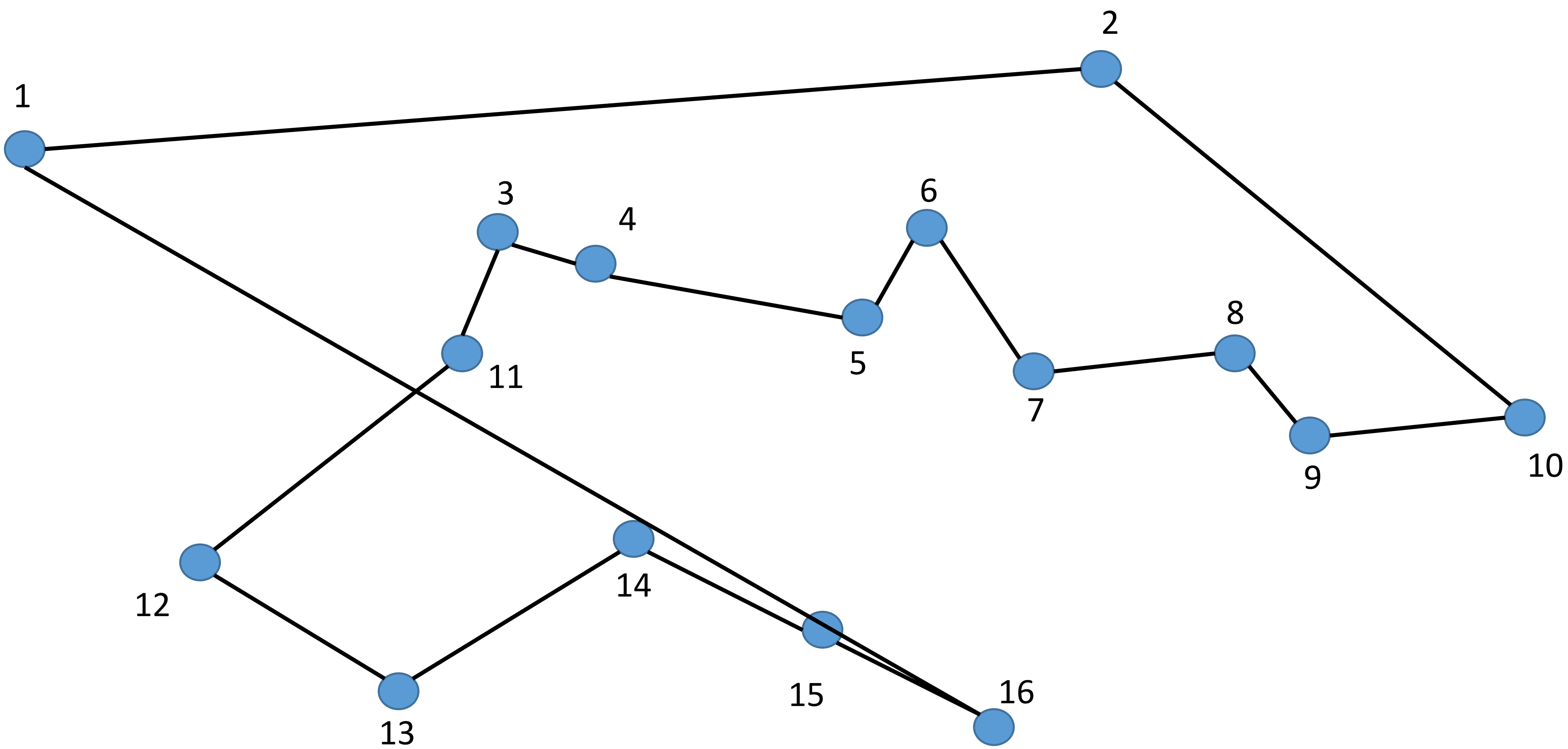


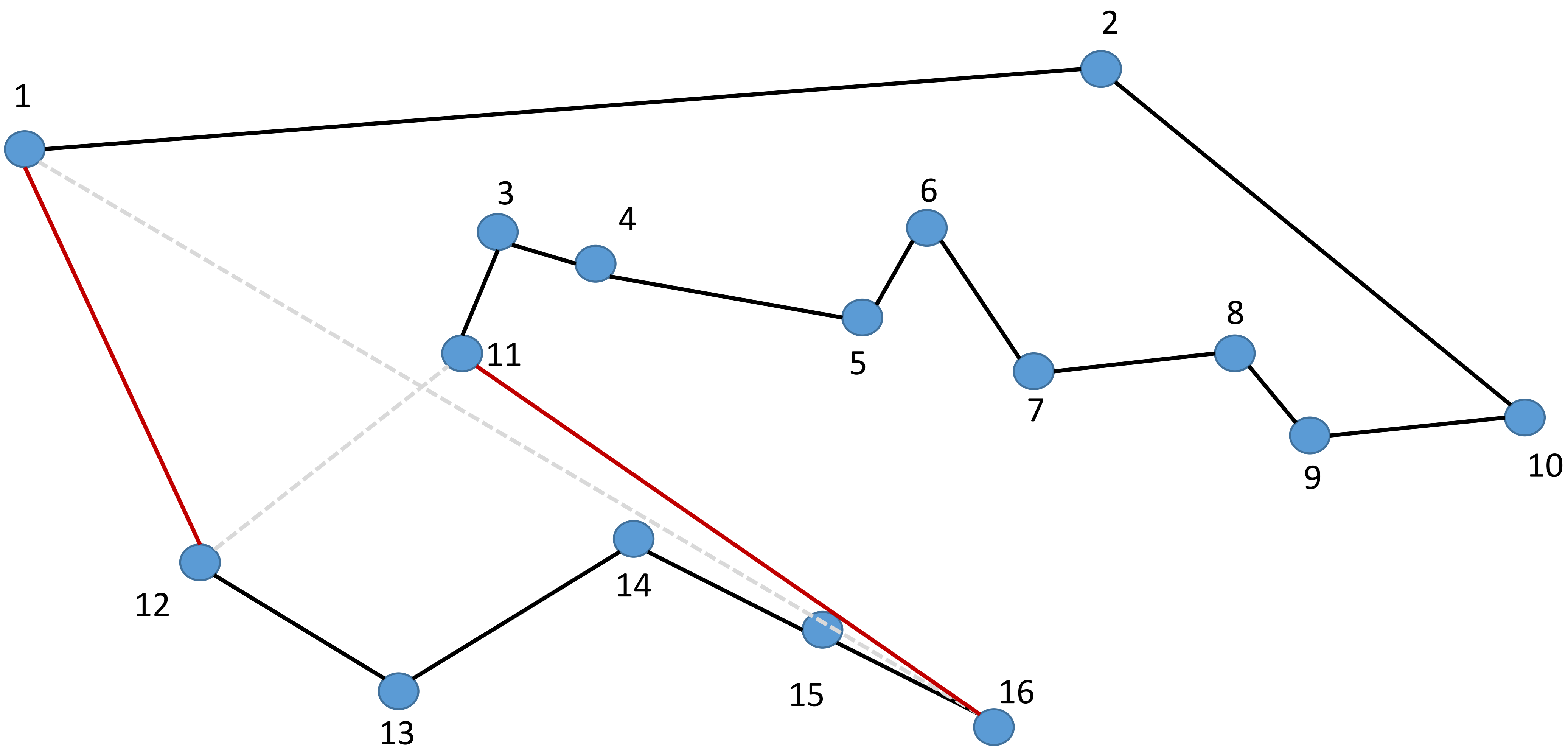
# Example

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **10** | **9** | **8** | **7** | **6** | **5** | **4** | **3** | **11** | **12** | **13** | **14** | **15** | **16** |



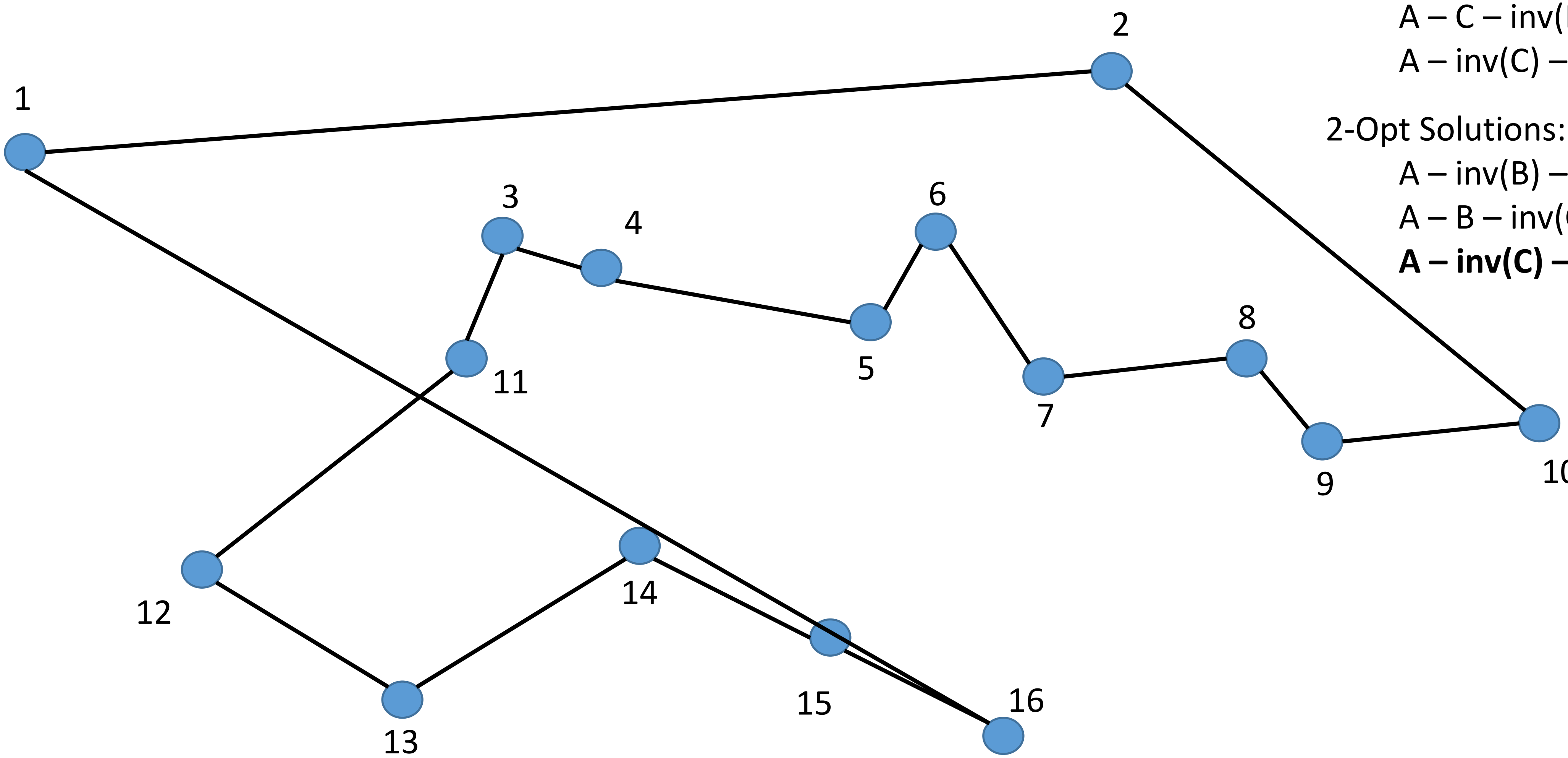
# Optimal Solution

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **10** | **9** | **8** | **7** | **6** | **5** | **4** | **3** | **11** | **12** | **13** | **14** | **15** | **16** |



# 3-Opt Operator

| | | | | A | | | | | | | | B | | C | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **10** | **9** | **8** | **7** | **6** | **5** | **4** | **3** | **11** | **12** | **13** | **14** | **15** | **16** |




3-Opt Solutions:

- A – inv(B) – inv(C)
- A – C – B
- A – C – inv(B)
- A – inv(C) – B

2-Opt Solutions:

- A – inv(B) – C
- A – B – inv(C)
- **A – inv(C) – inv(B)**

# 3-Opt Operator

| A | | | | | | | | | | | B | | | C | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **10** | **9** | **8** | **7** | **6** | **5** | **4** | **3** | **11** | **12** | **13** | **14** | **15** | **16** |

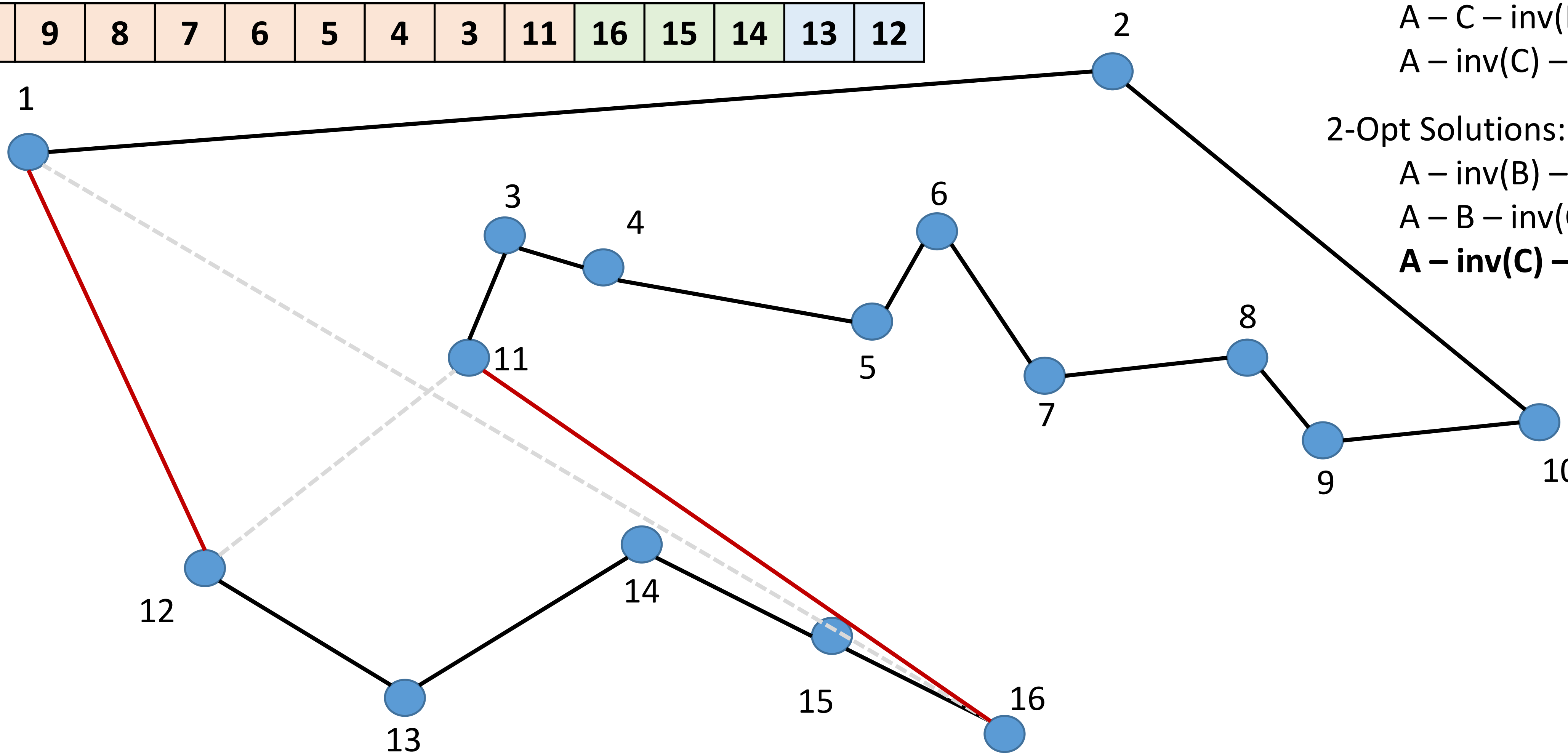
| A | | | | | | | | | | | rev(C) | | | rev(B) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **10** | **9** | **8** | **7** | **6** | **5** | **4** | **3** | **11** | **16** | **15** | **14** | **13** | **12** |

3-Opt Solutions:

- A – inv(B) – inv(C)
- A – C – B
- A – C – inv(B)
- A – inv(C) – B

2-Opt Solutions:

- A – inv(B) – C
- A – B – inv(C)
- **A – inv(C) – inv(B)**



# Swap Operator

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **10** | **9** | **8** | **7** | **6** | **5** | **4** | **3** | **11** | **12** | **13** | **14** | **15** | **16** |

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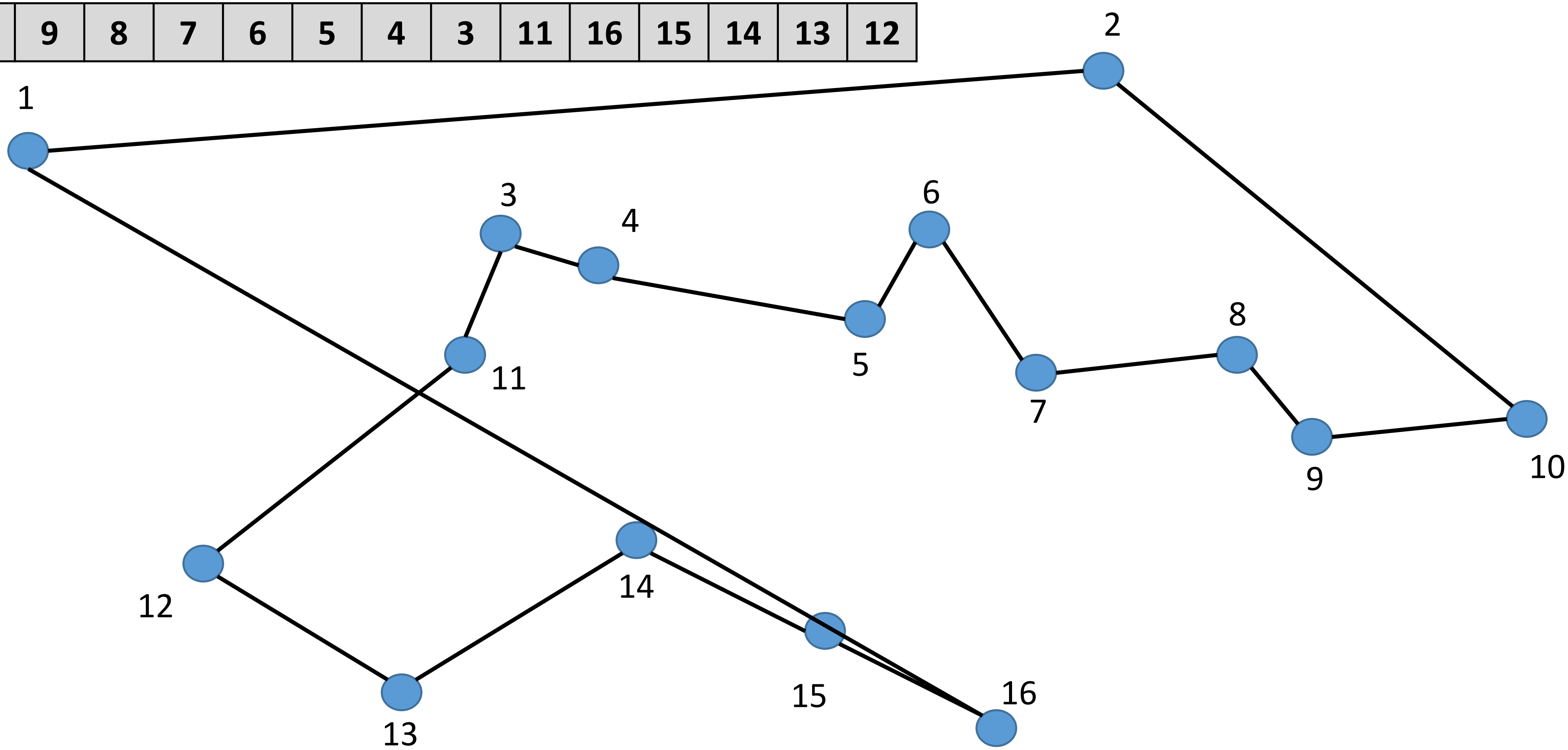
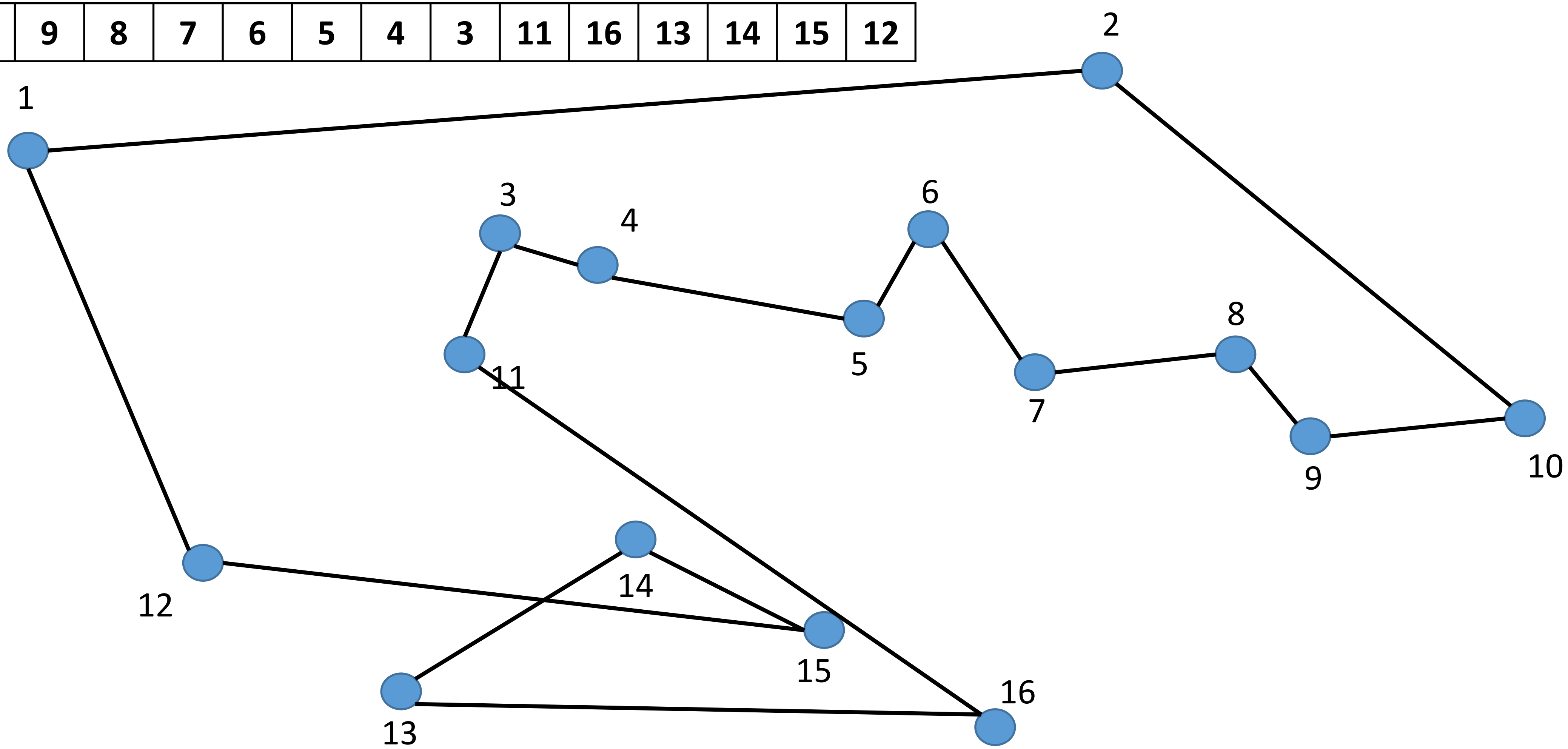
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **10** | **9** | **8** | **7** | **6** | **5** | **4** | **3** | **11** | **16** | **15** | **14** | **13** | **12** |



# Swap Operator

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 11 | 12 | 13 | 14 | 15 | 16 |
| 1 | 2 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 11 | 16 | 13 | 14 | 15 | 12 |



# Swap Operator

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **10** | **9** | **8** | **7** | **6** | **5** | **4** | **3** | **11** | **16** | **13** | **14** | **15** | **12** |

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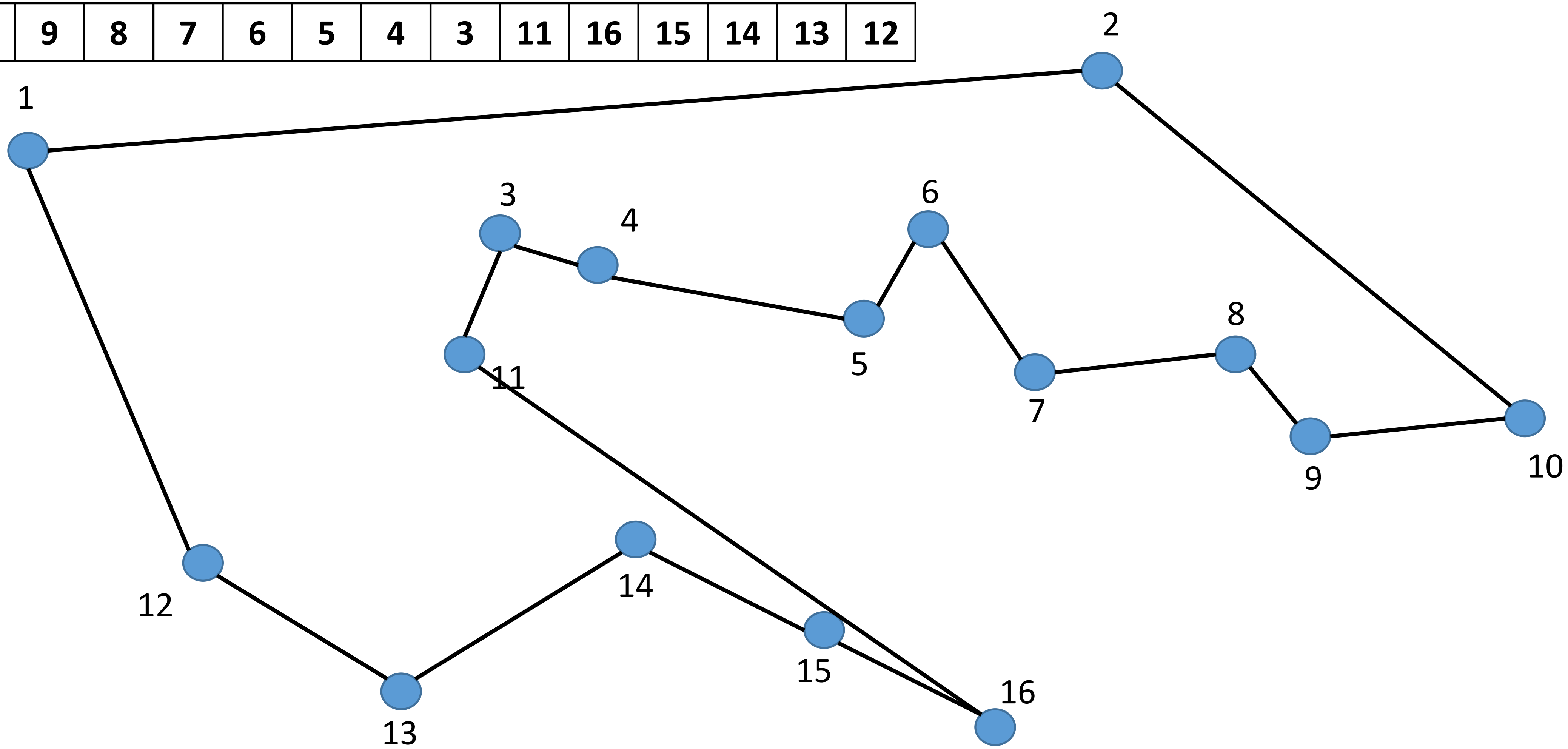
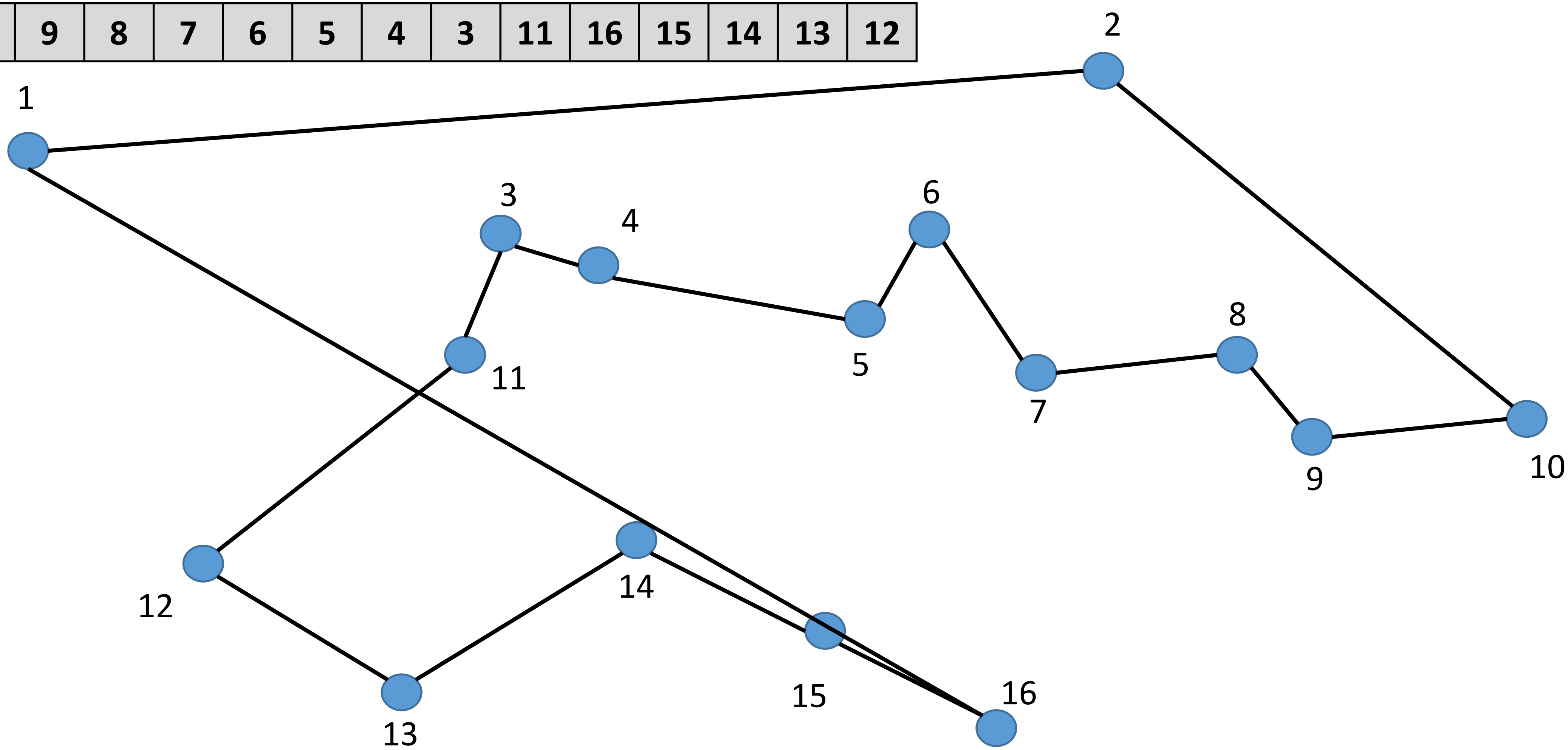
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **10** | **9** | **8** | **7** | **6** | **5** | **4** | **3** | **11** | **16** | **15** | **14** | **13** | **12** |



# Insertion Operator

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 11 | 12 | 13 | 14 | 15 | 16 |

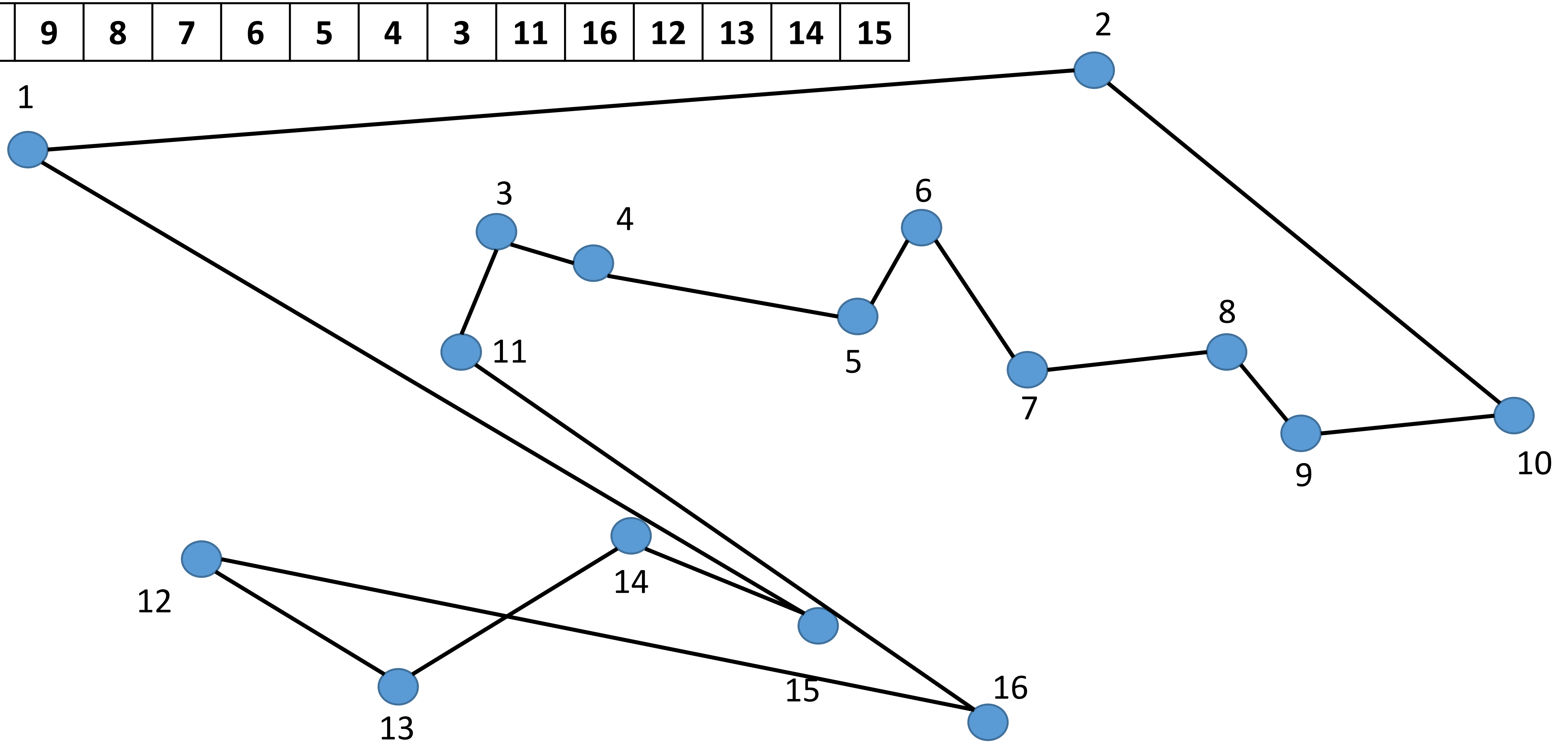
| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 11 | 16 | 15 | 14 | 13 | 12 |



# Insertion Operator

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 11 | 12 | 13 | 14 | 15 | 16 |

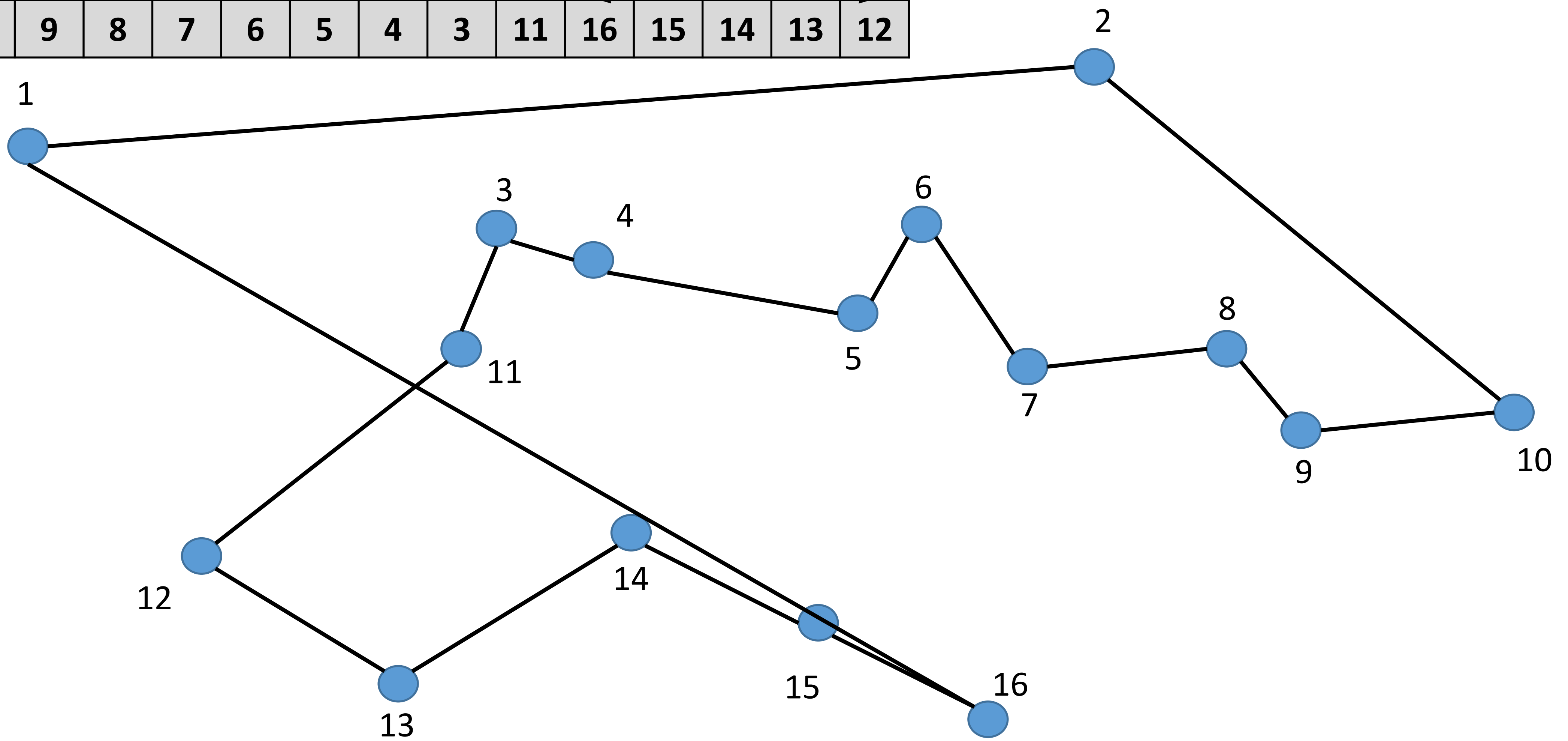
| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 11 | 16 | 12 | 13 | 14 | 15 |



# Inversion Operator

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 11 | 12 | 13 | 14 | 15 | 16 |

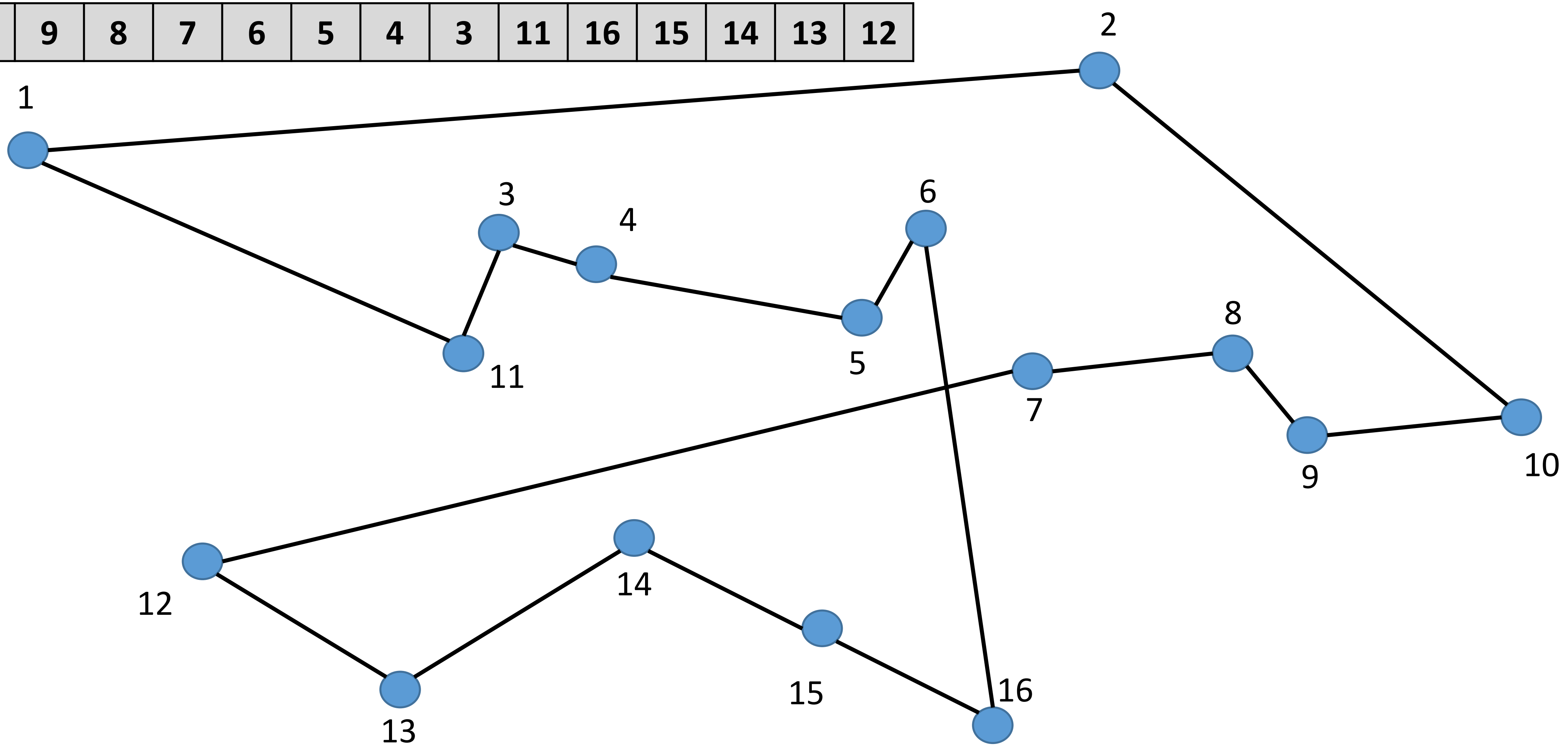
| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 11 | 16 | 15 | 14 | 13 | 12 |



# Inversion vs 3-Opt Operator

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 9 | 8 | 7 | 12 | 13 | 14 | 15 | 16 | 6 | 5 | 4 | 3 | 11 |

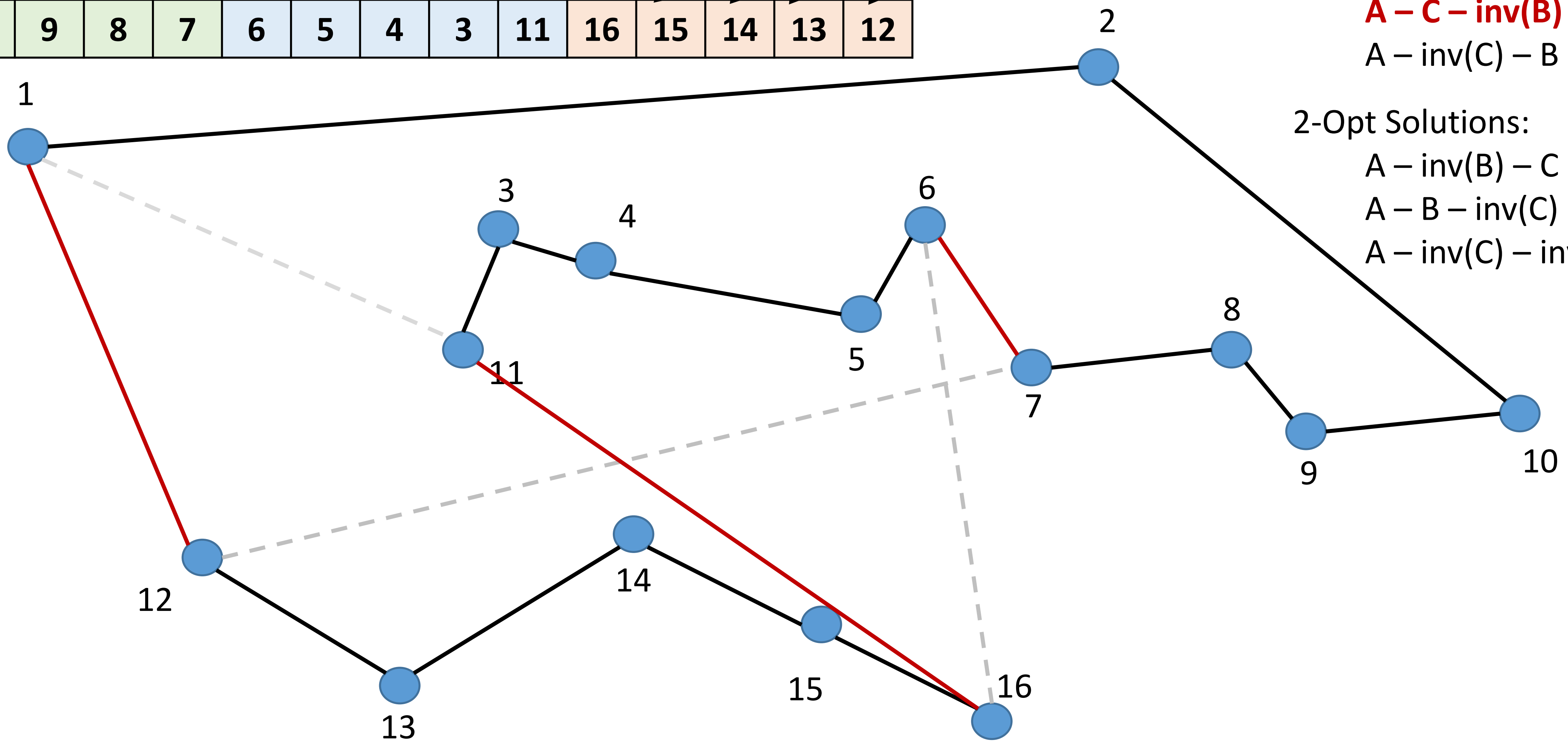
| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 11 | 16 | 15 | 14 | 13 | 12 |



# Inversion vs 3-Opt Operator

| A | | | | | | B | | | | | C | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **10** | **9** | **8** | **7** | **12** | **13** | **14** | **15** | **16** | **6** | **5** | **4** | **3** | **11** |

| A | | | | | | C | | | | | rev(B) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **10** | **9** | **8** | **7** | **6** | **5** | **4** | **3** | **11** | **16** | **15** | **14** | **13** | **12** |




3-Opt Solutions:

- A – inv(B) – inv(C)
- A – C – B
- **A – C – inv(B)**
- A – inv(C) – B

2-Opt Solutions:

- A – inv(B) – C
- A – B – inv(C)
- A – inv(C) – inv(B)

# K-Opt Operator in Python

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# TSP Example

- **Solution Representation:** Premutation Encoding
- **Search Operators:** Swap 2 locations ; Insertion ; 3-Opt
- **Replacement Procedure:** First Descent ; First Descent ; Best Descent

***Number of candidate locations n=100***

# TSP – Hill Climbing Generation Phase

```python
#Exchange Operator
def swap_random(seq):
    idx = range(len(seq))
    i1, i2 = random.sample(idx, 2)
    seq[i1], seq[i2] = seq[i2], seq[i1]
```

**Select two random locations**
**Swap location in the permutation**

```python
#Insert Operator
def insert_random(seq):
    idx = range(len(seq))
    i1=random.sample(idx, 1)
    remove_index=np.where(np.array(seq)==i1)
    seq.pop(int(remove_index[0]))
    seq.insert(random.sample(idx, 1)[0], i1[0])
```

**Select location to insert (i1)**
**Remove from permutation list**
**Insert in a new position in the permutation list**

```python
#3-Opt
def k_opt(seq):
    global Solution_i
    idx = range(len(seq))
    i1=np.sort(random.sample(idx, 2))

    #Split in 3
    Opt1=seq[0:i1[0]]
    Opt2=seq[(i1[0]):i1[1]]
    Opt3=seq[(i1[1]):len(seq)]
    Opt2rev=Opt2[::-1]
    Opt3rev=Opt3[::-1]

    #3-Opt Solutions
    Sol1=Opt1+Opt2rev+Opt3rev
    Sol2=Opt1+Opt3+Opt2
    Sol3=Opt1+Opt3+Opt2rev
    Sol4=Opt1+Opt3rev+Opt2

    #2 Opt Solution
    Sol5=Opt1+Opt2rev+Opt3
    Sol6=Opt1+Opt2+Opt3rev
    Sol7=Opt1+Opt3rev+Opt2rev

    OptNeigh=[Sol1,Sol2,Sol3,Sol4,Sol5,Sol6,Sol7]

    ObjValue_Neigh=list();
    #Compute Obj Value of All Solutions
    for i_index in range(len(OptNeigh)):
        Solution_Neigh=OptNeigh[i_index]

        dfSolution_i=pd.DataFrame(Solution_Neigh)
        dflinkindex_p1=dfSolution_i
        dflinkindex_p2=dfSolution_i.shift(-1)
        dflinkindex_p2.loc[n-1]=dflinkindex_p1.loc[0]
        linkindex_p1=dflinkindex_p1.to_numpy()
        linkindex_p2=dflinkindex_p2.to_numpy()
        linkindex_p1=linkindex_p1.astype(int)
        linkindex_p2=linkindex_p2.astype(int)
        linkindex_p1=linkindex_p1.transpose()[0]
        linkindex_p2=linkindex_p2.transpose()[0]

        #Compute Objective Value
        ObjValue=sum(distancelct[linkindex_p1,linkindex_p2])
        ObjValue_Neigh=np.append(ObjValue_Neigh,ObjValue)
    OptNeigh[np.argmin(ObjValue_Neigh)]
    Solution_i=OptNeigh[np.argmin(ObjValue_Neigh)]
```

**Follow the procedure explained in slide 43**

**Explore the whole neighbourhood (Best Descent) and select the best solution**

# TSP – Hill Climbing Generation Phase

```
random.seed(3)
iteration=0
ObjValueOpt=ObjValue
Objvalue_list=ObjValue
program_starts = time.time()
cputime_i=[0,0]
OptSolution=Solution_i

while cputime_i[-1]<6000:

    iteration=iteration+1
    Solution_i=copy.deepcopy(OptSolution)

    swap_it=0
    while swap_it<no_swap:
        swap_random(Solution_i)
        swap_it=swap_it+1

    dfSolution_i=pd.DataFrame(Solution_i)
    dfSolution_i
    dflinkindex_p1=dfSolution_i
    dflinkindex_p2=dfSolution_i.shift(-1)
    dflinkindex_p2.loc[n-1]=dflinkindex_p1.loc[0]
    linkindex_p1=dflinkindex_p1.to_numpy()
    linkindex_p2=dflinkindex_p2.to_numpy()
    linkindex_p1=linkindex_p1.astype(int)
    linkindex_p2=linkindex_p2.astype(int)
    linkindex_p1=linkindex_p1.transpose()[0]
    linkindex_p2=linkindex_p2.transpose()[0]

    #Compute Objective Value
    ObjValue=sum(distancelct[linkindex_p1,linkindex_p2])
```

**Apply search operator**

**Generation Phase**

**(Search Operator Code)**

**Compute Objective Value**

# TSP – Hill Climbing Replacement Phase

```python
    #Update Optimal Solution
    if ObjValue<ObjValueOpt:
        ObjValueOpt=copy.deepcopy(ObjValue)
        OptSolution=copy.deepcopy(Solution_i)

    Objvalue_list=np.append(Objvalue_list, ObjValueOpt)
    now = time.time()
    cputime_i=np.append(cputime_i, now-program_starts)

        #def connectpoints(x,y,p1,p2):
        #    x1, x2 = x[p1], x[p2]
        #    y1, y2 = y[p1], y[p2]
        #    plt.plot([x1,x2],[y1,y2],'k-')

        #for i_index in range(len(linkindex_p2)):
        #    connectpoints(coordlct_x,coordlct_y,linkindex_p1[i_index],linkindex_p2[i_index])

        #plt.plot(coordlct_x, coordlct_y, 'o', color='black');

        #clear_output(wait=True)
        #plt.draw()
        #plt.pause(0.1)
        #plt.clf()

#Update last objective value
Objvalue_list=np.append(Objvalue_list, min(Objvalue_list))
now = time.time()
cputime_i=np.append(cputime_i, now-program_starts)
```

**Hill Climbing**

**If objective value is worse than the best objective value found in previous iterations, then nothing happens; otherwise we update the best solution**

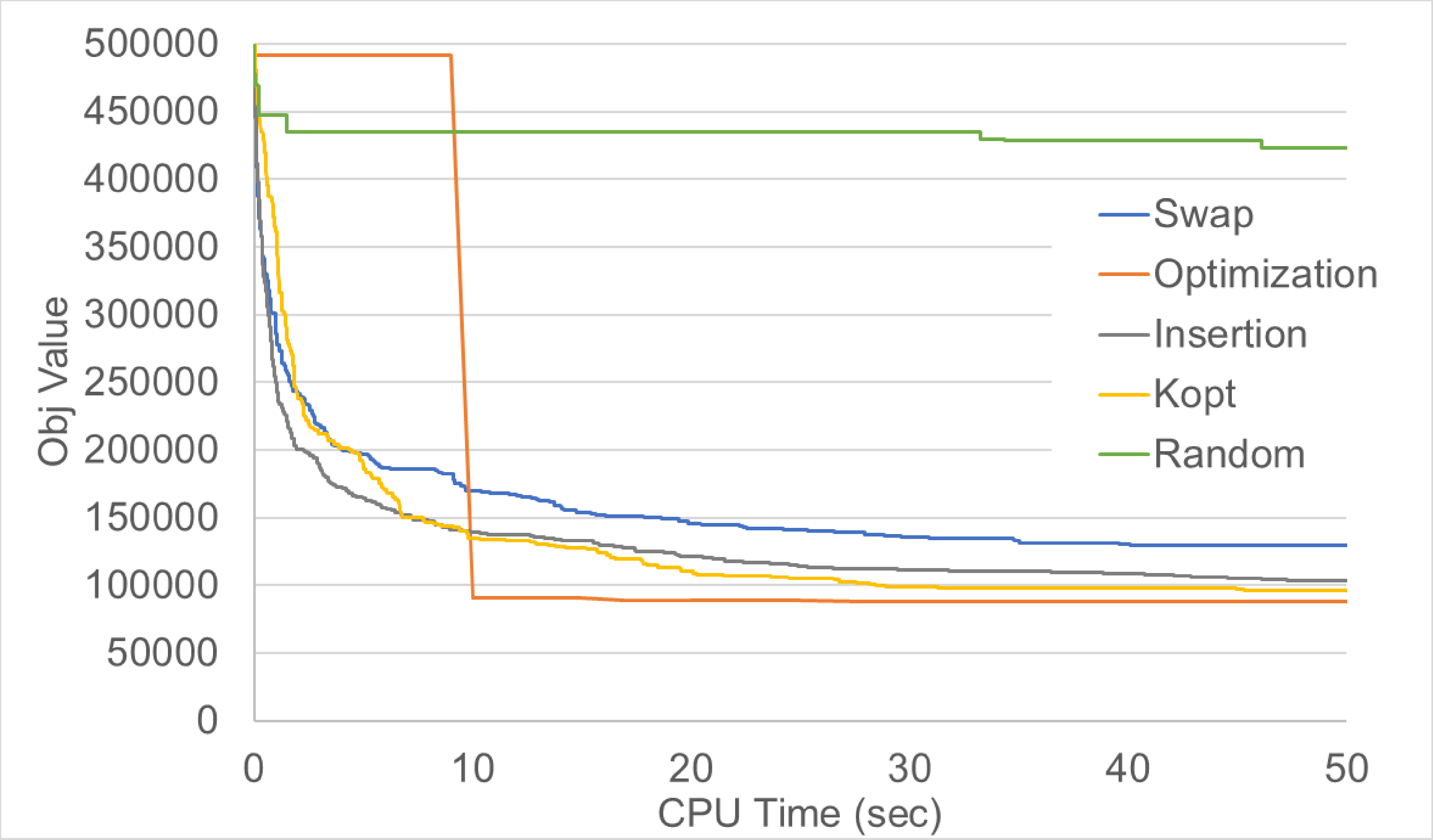
**Plot**

**Replacement Phase**

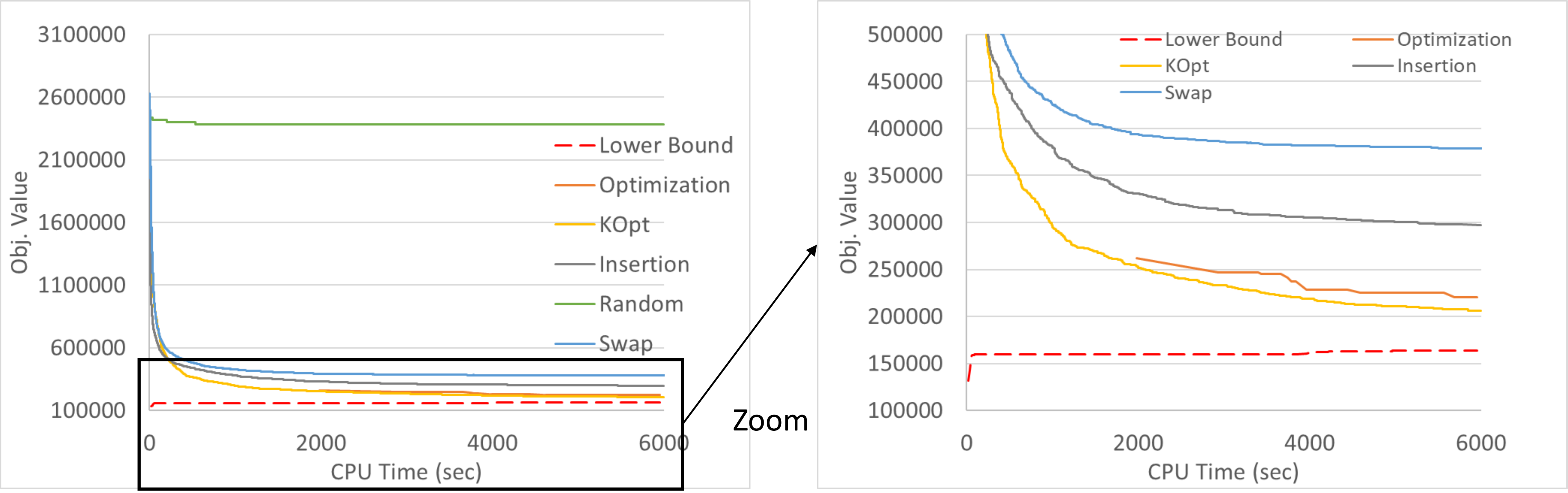
**(First Descent)**

**Keep trace of the objective values obtained over time**

# TSP Solution (n=100)



# TSP Solution (n=500)



# Constraint Handling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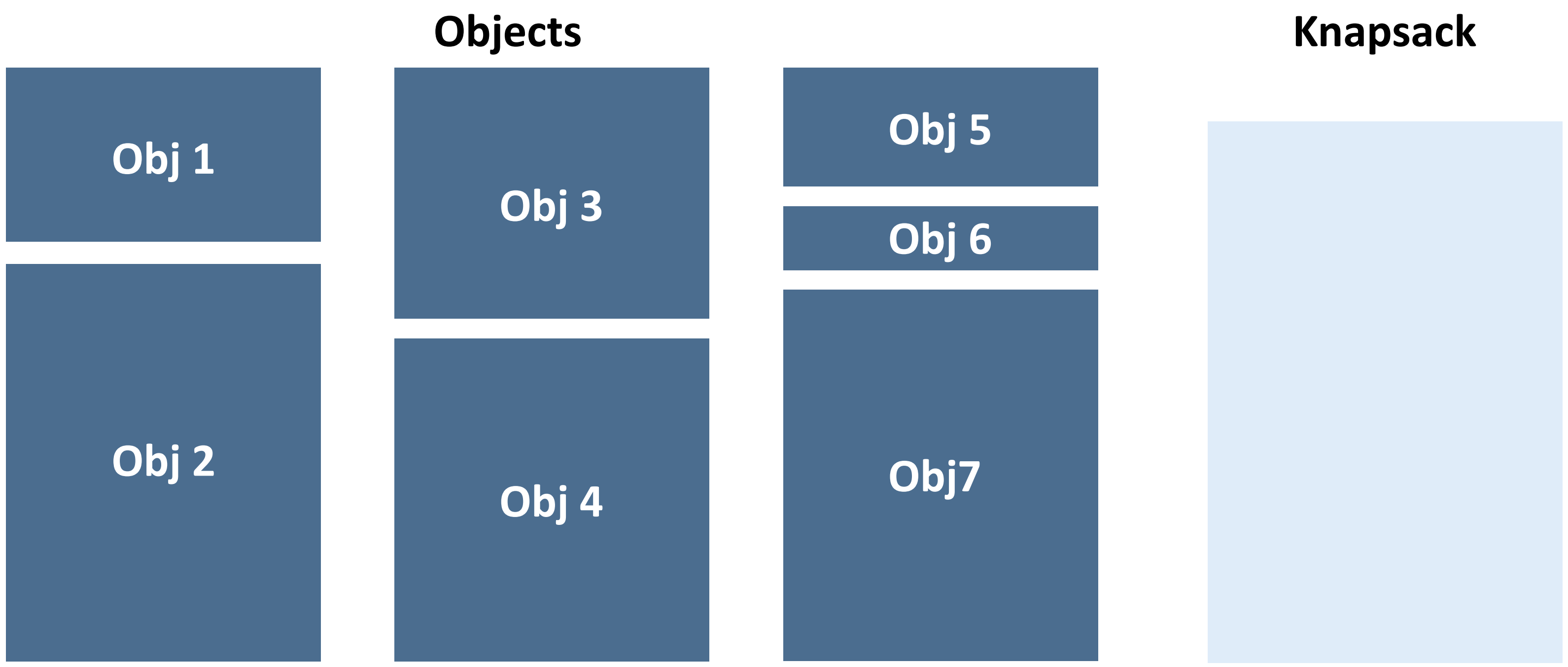
# Constraint Handling

- Dealing with constraints in optimization problems is an important topic for the efficient design of metaheuristics.
- Indeed, many continuous and discrete optimization problems are constrained, and it is not trivial to deal with those constraints.

**Example Knapsack Problem:**




**Solution Encoding**

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Search Operator**

Flip n bits of the solution

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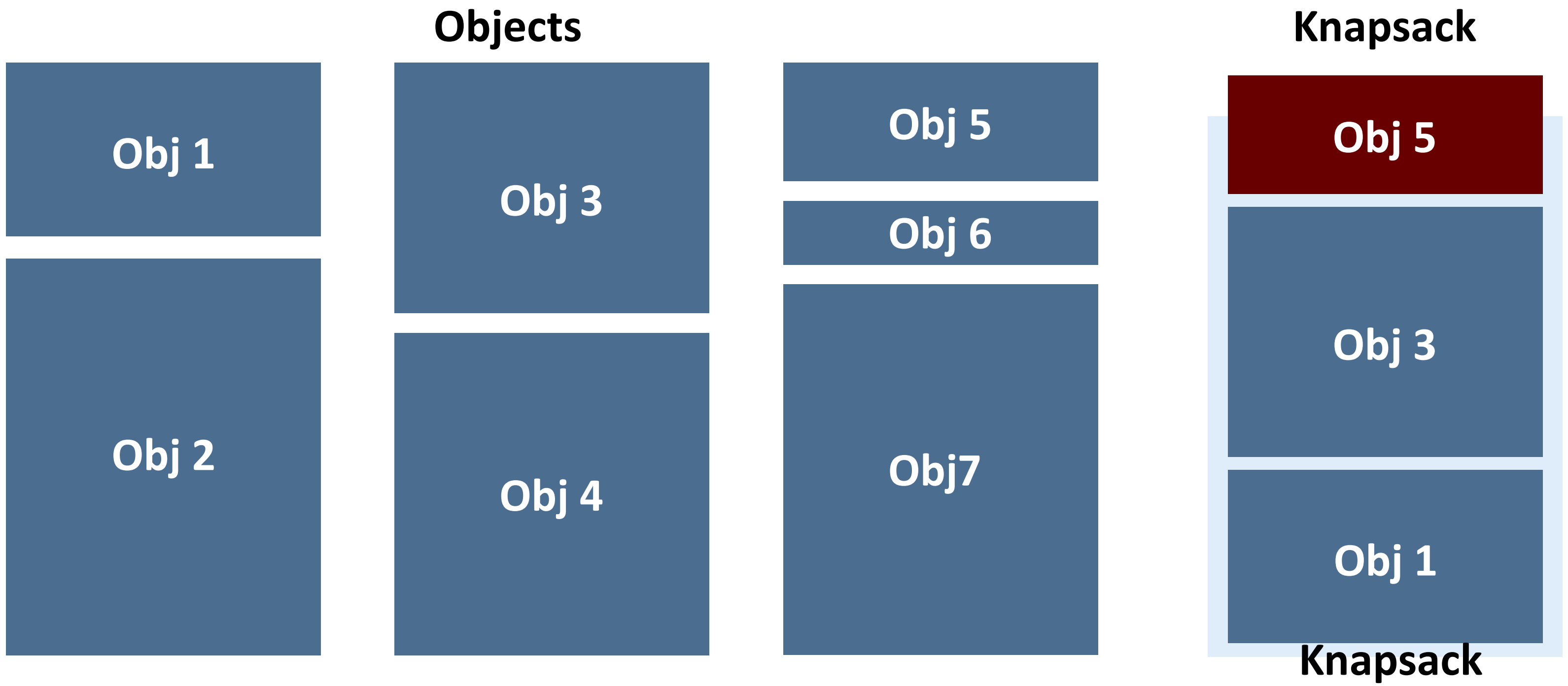


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| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 0 | 0 |

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**Infeasible Solution**

# Constraint Handling Techniques

- **Reject Strategies:** represent a simple approach, where **only feasible solutions are kept during the search** and then infeasible solutions are automatically discarded. This kind of strategies are conceivable if the portion of infeasible solutions of the search space is very small.
- **Repairing strategies:** A **repairing procedure** is applied to infeasible solutions to generate feasible ones (e.g. extracting from the knapsack some elements to satisfy the capacity constraint in the knapsack problem)
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- **Preserving Strategies:** In preserving strategies for constraint handling, a **specific representation and operators will ensure the generation of feasible solutions.** They incorporate problem-specific knowledge into the representation and search operators to generate only feasible solutions

# Reject Strategies

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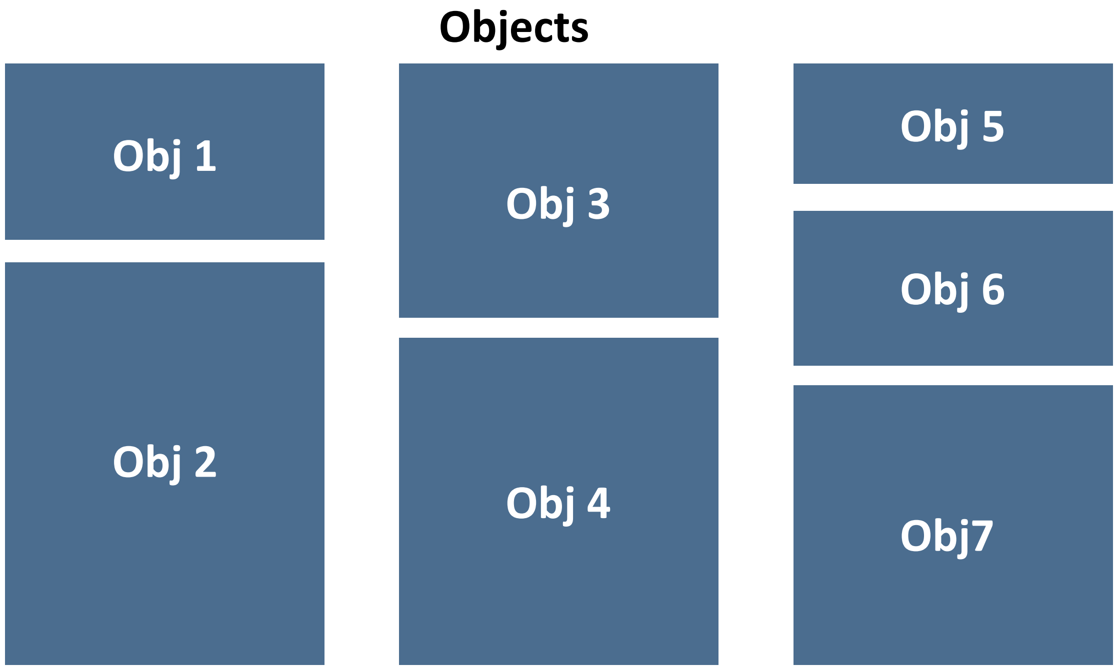


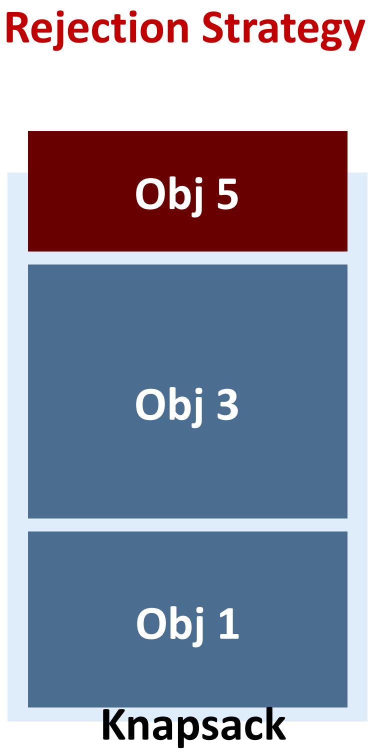


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# Repairing strategies

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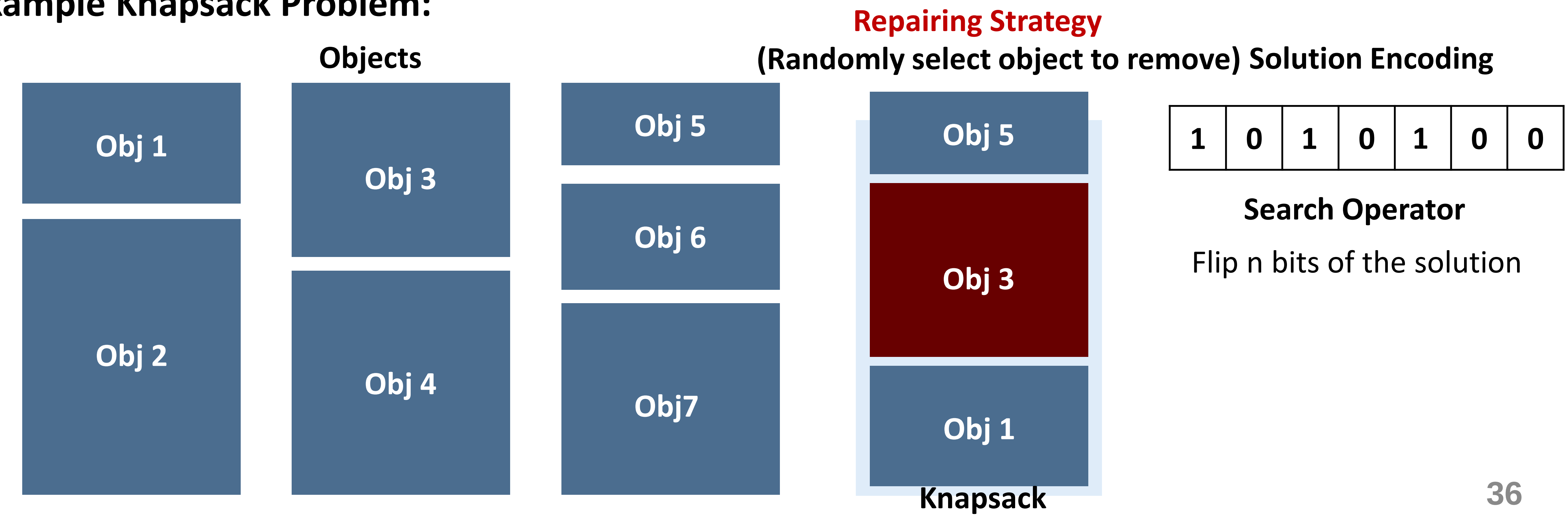

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# Penalizing Strategies

- The objective function *f* may be penalized in a linear manner, where *c(s)* represents the cost of the constraint violation and *λ* the weights given to infeasibilities.

$$f'(s) = f(s) + \lambda c(s)$$

- Different penalty functions may be use:
  - **Violated constraints**: A straightforward function is to **count the number of violated constraints**. No information is used on how close the solution is to the feasible region of the search space. (e.g. number of bins with capacity violated in the bin-packing problem)
  - **Amount of infeasibility**: Information on **how close a solution is to a feasible region** is taken into account (e.g. how much the capacity of a bin is exceeded in the bin-packing problem).

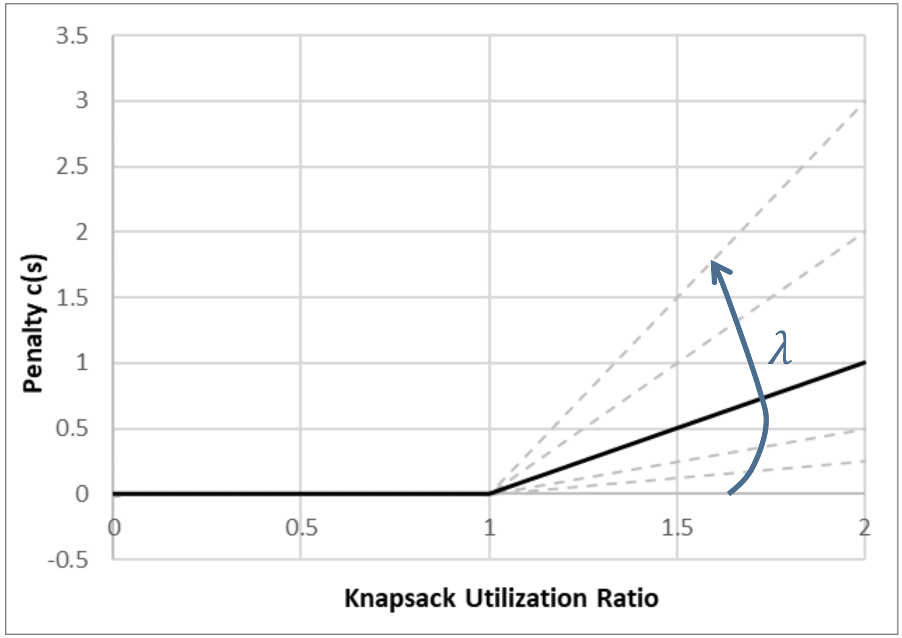
# Penalizing Strategies

| | $w_i$ | $f_i$ |
|---|---|---|
| Obj 1 | 2 | 5 |
| Obj 2 | 3.75 | 7 |
| Obj 3 | 2.5 | 3 |
| Obj 4 | 3 | 5 |
| Obj 5 | 1 | 4 |
| Obj 6 | 1.5 | 8 |
| Obj 7 | 2.75 | 7 |
| cap | 4 | |

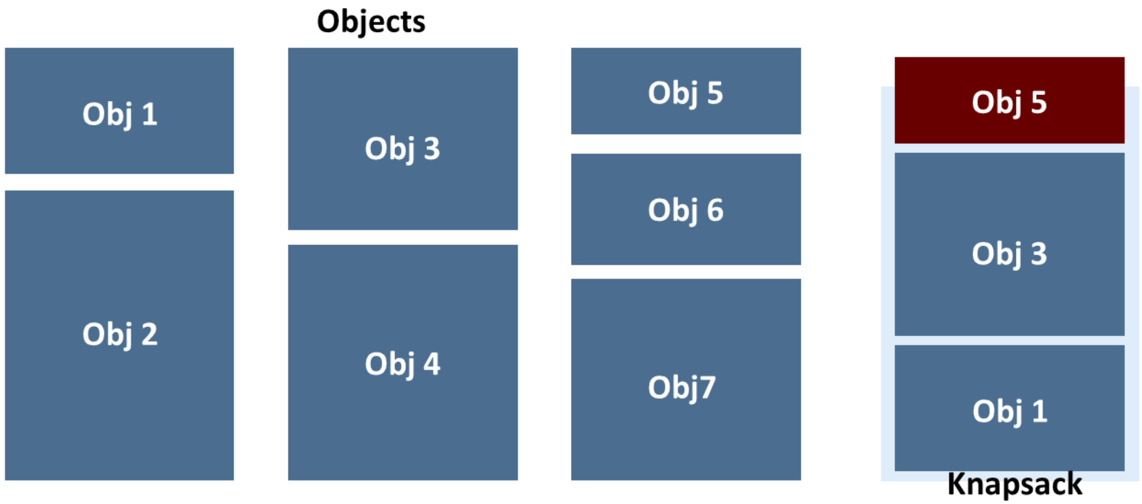
$$f'(s) = f(s) + \lambda c(s)$$

$$c(s) = -\frac{min(0, \sum_i w_i - Cap)}{Cap}$$

$$f(s) = \sum_i f_i$$




**Example Knapsack Problem:**




5.5 kg

$$\lambda = 1$$

$$\lambda c(s) = -\frac{|(2 + 2.5 + 1 - 4)|}{4} = -0.375$$

$$f(s) = 5 + 3 + 4 = 12$$

$$f'(s) = \mathbf{11.625}$$

**Infeasible Solution**

# Penalizing Strategies

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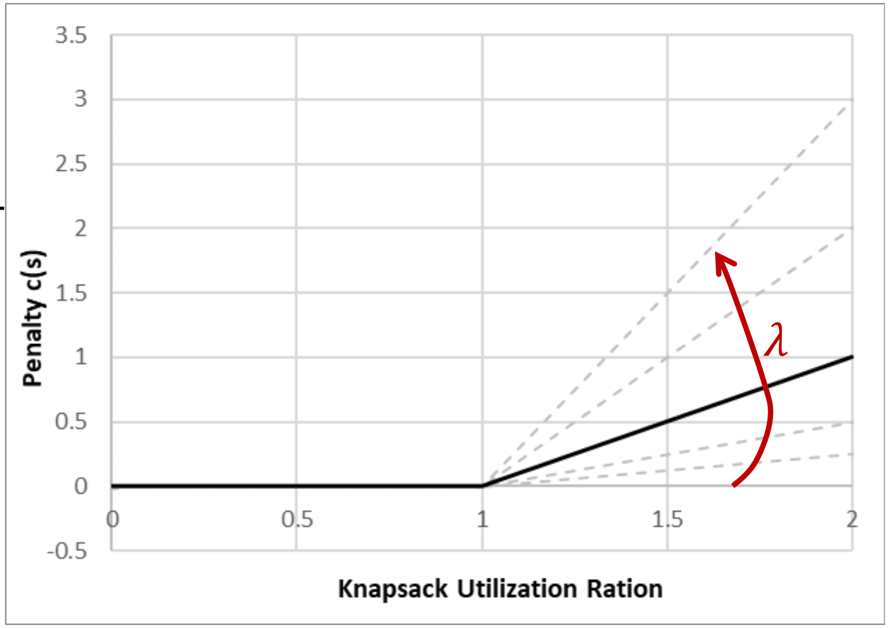


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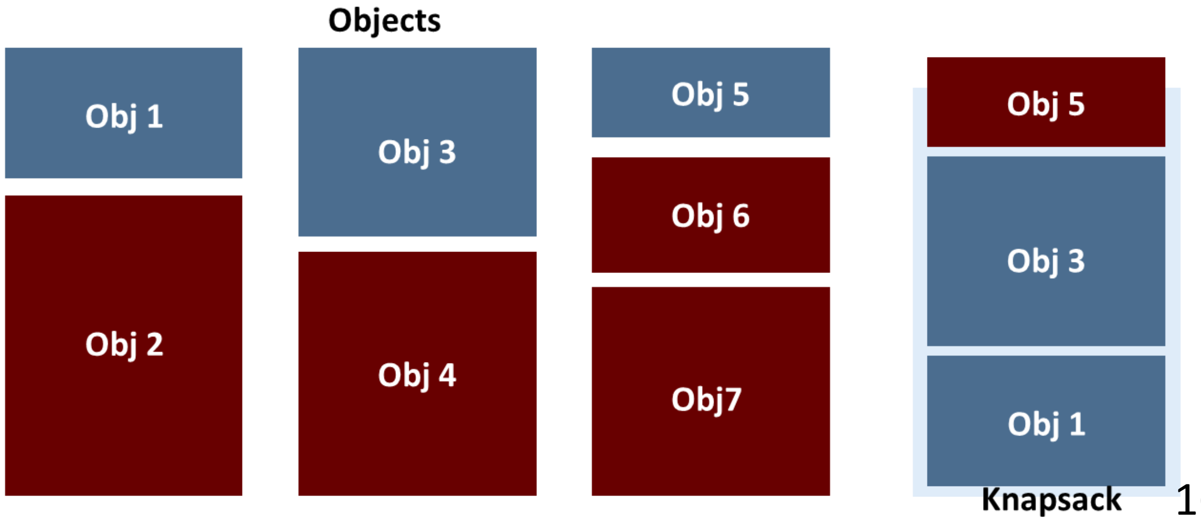


16.5 kg

$$\lambda c(s) = -4.125$$

$$f(s) = 39$$

$$f'(s) = 34.875$$

**Infeasible Solution**

# Penalizing Strategies

| | $w_i$ | $f_i$ |
|---|---|---|
| Obj 1 | 2 | 5 |
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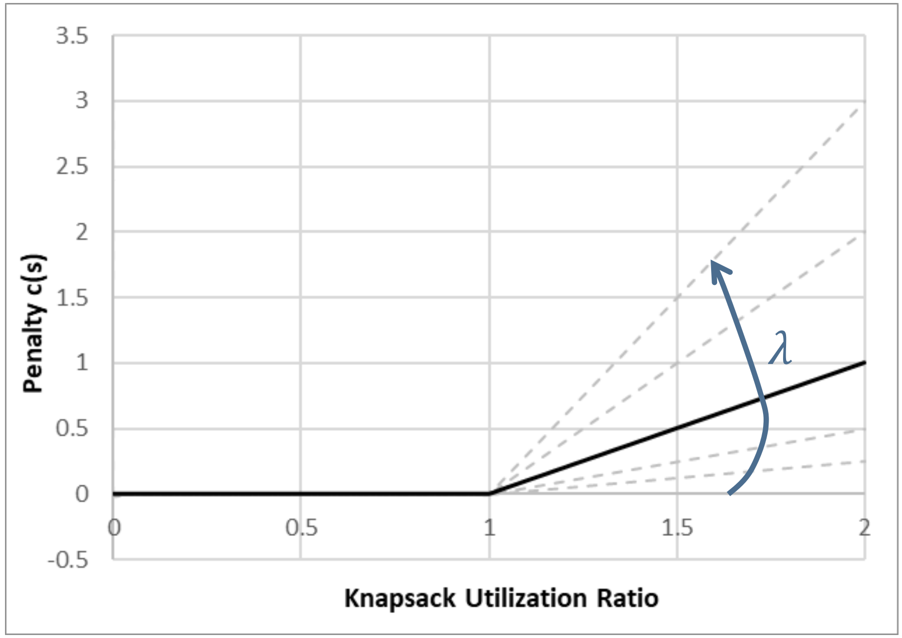


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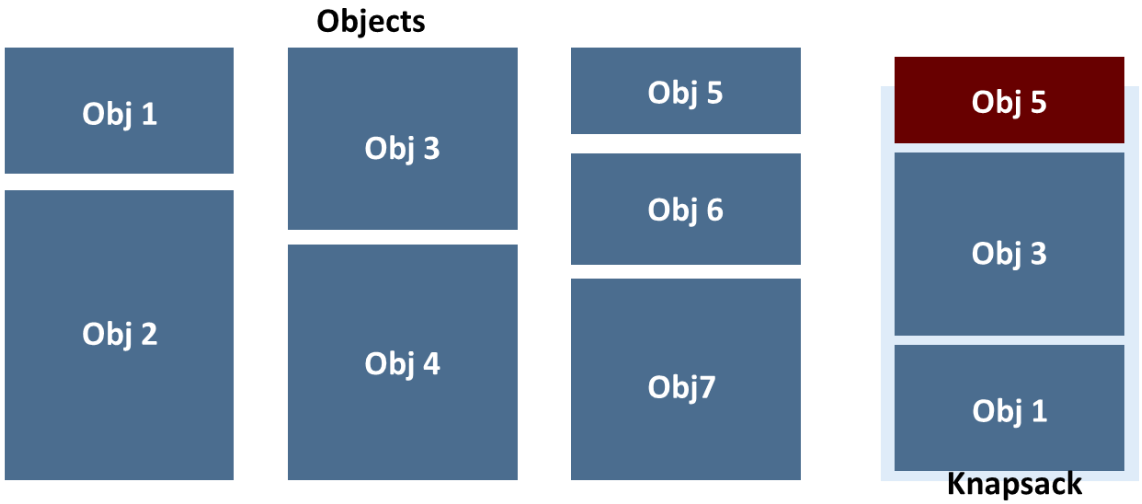


5.5 kg

$$\lambda = 10$$

$$\lambda c(s) = -3.75$$

$$f(s) = 12$$

$$f'(s) = \mathbf{8.25}$$

**Infeasible Solution**

# Penalizing Strategies

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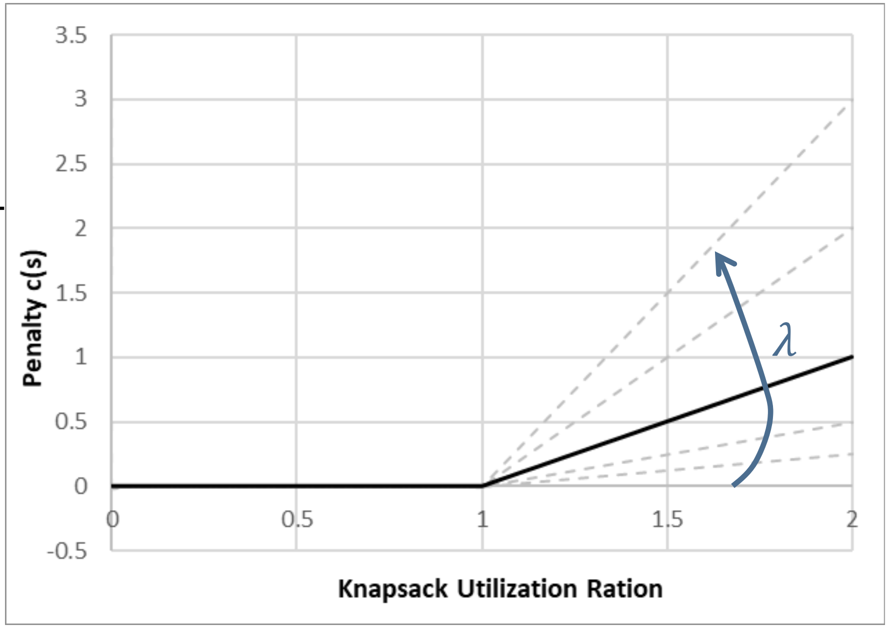


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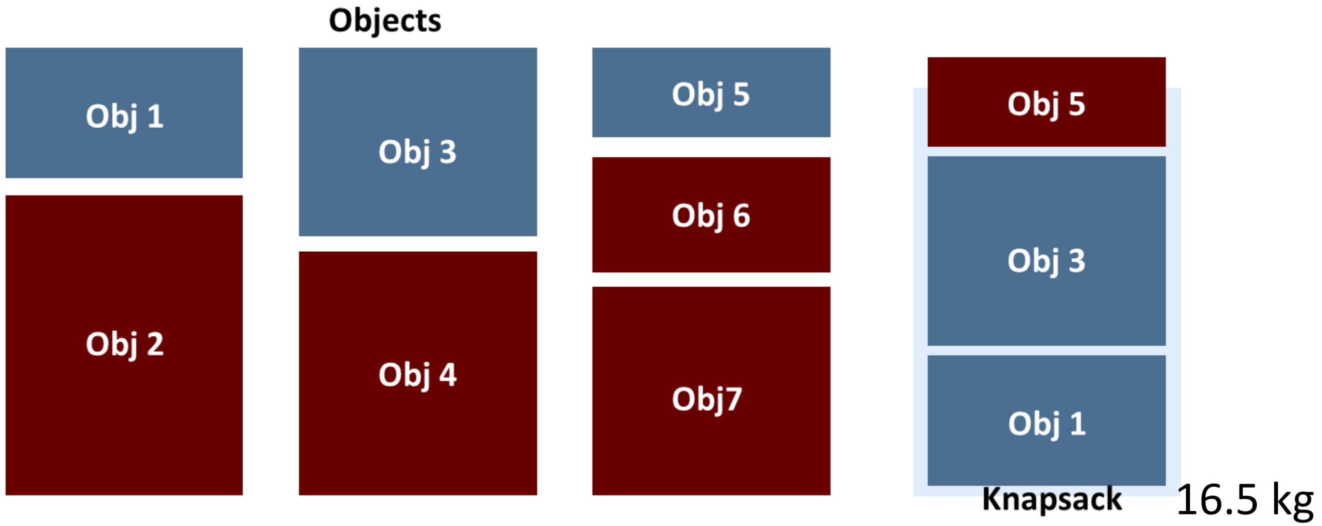


$$\lambda = 10$$

$$\lambda c(s) = -41.25$$

$$f(s) = 39$$

$$f'(s) = \mathbf{-2.25}$$

**Infeasible Solution**

# Penalizing Strategies

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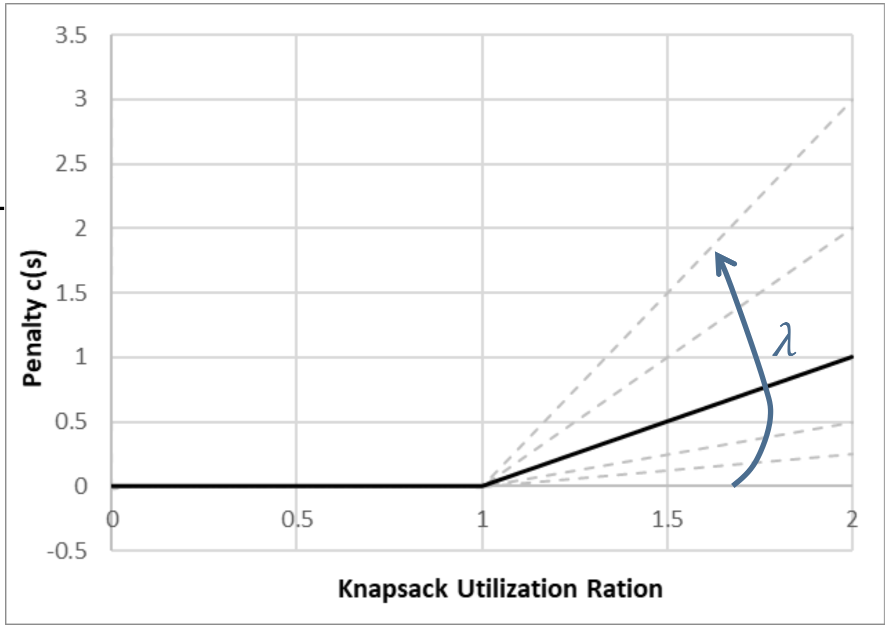


$$\lambda = 10$$

$$\lambda c(s) = -2.813$$

$$f(s) = 17$$

$$f'(s) = \mathbf{14.187}$$

**Infeasible Solution**

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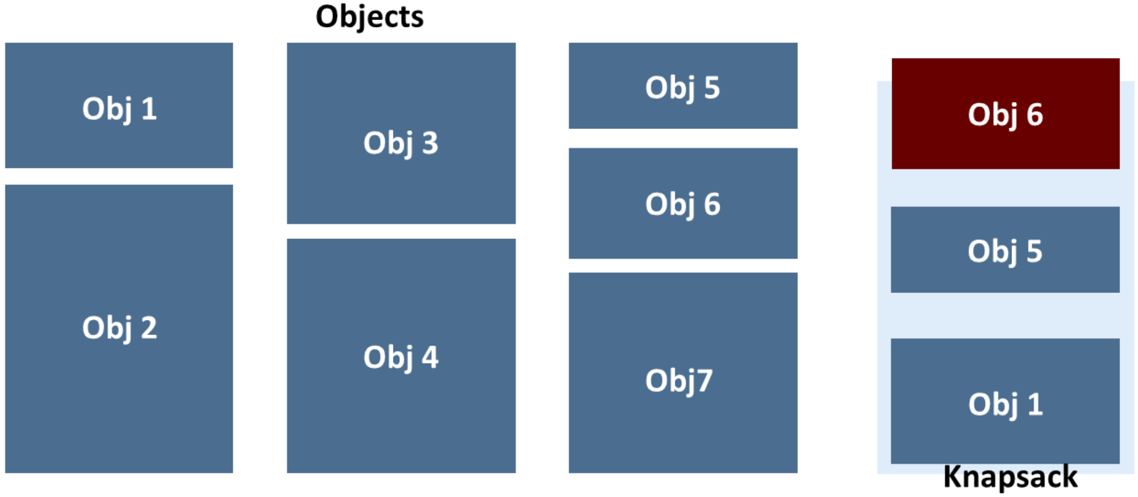


4.5 kg

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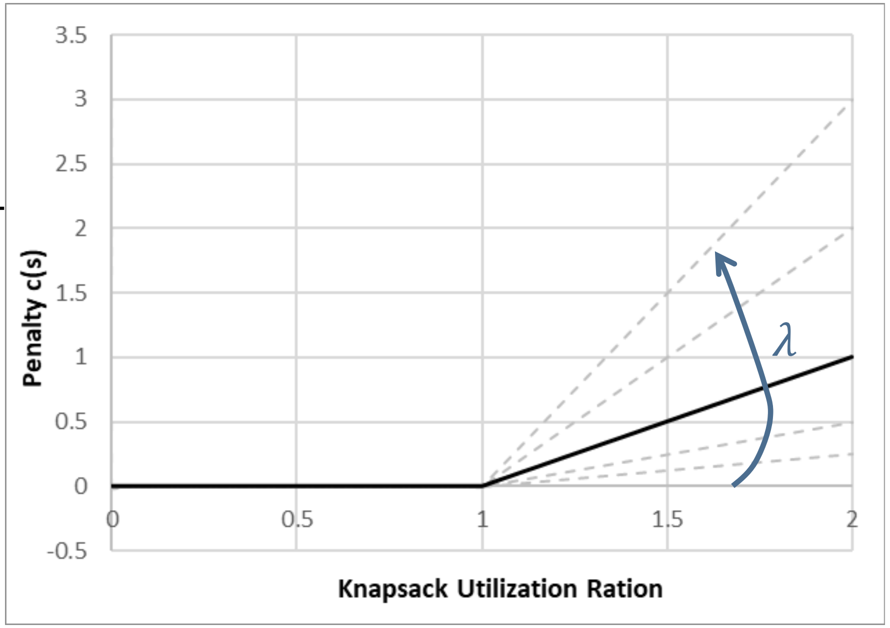


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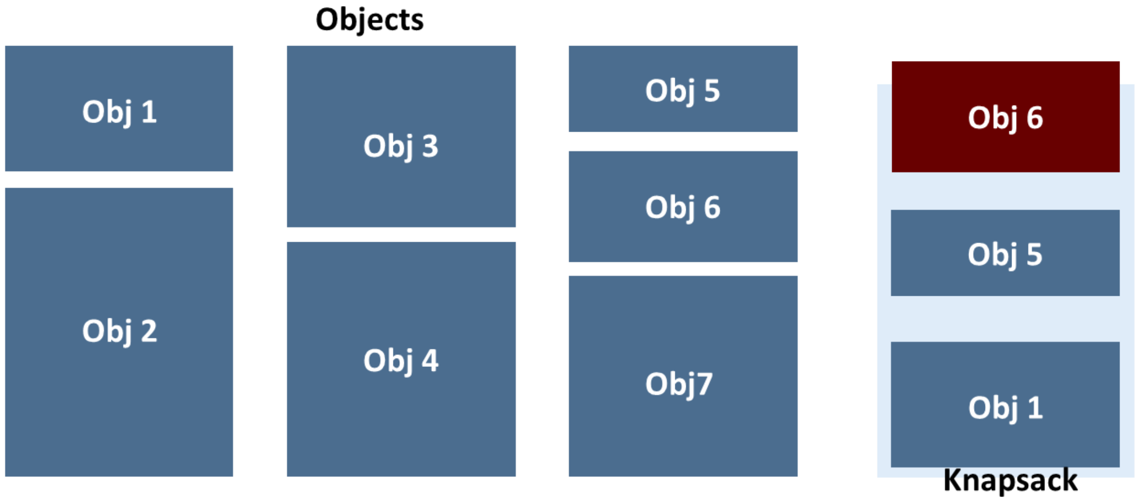



4.5 kg

$$\lambda c(s) = 0$$

$$f(s) = 5 + 1 + 8 = 13$$

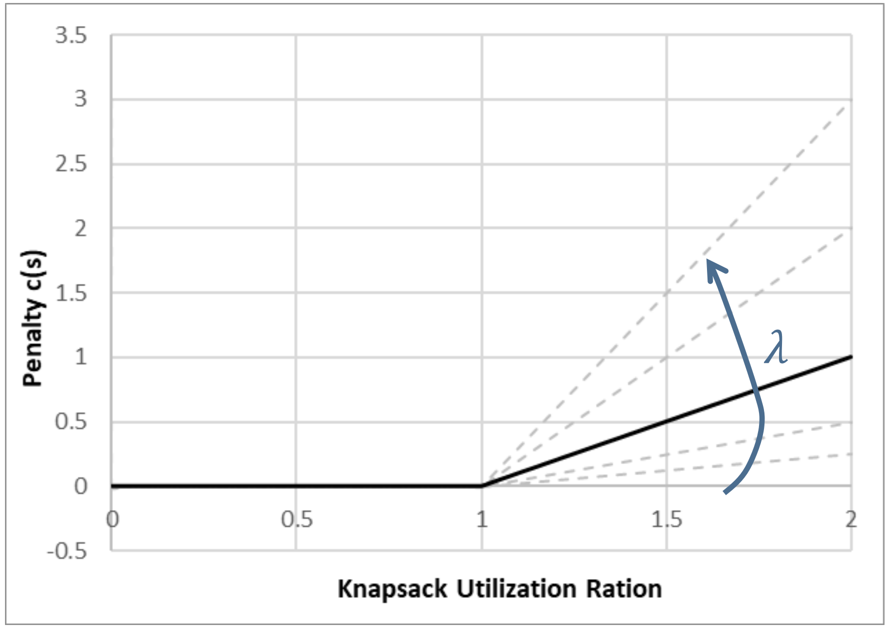
$$f'(s) = 13$$

Feasible solution, but worse objective value

# Adaptative Penalization

- The previously presented penalty functions do not exploit any information of the search process. In adaptive penalty functions, **knowledge on the search process is included to improve the efficiency and the effectiveness of the search.**

- Example:
  - The parameters $\lambda$ is self-adjusting. Initially, the parameter is initialized to 1.
  - The parameters $\lambda$ is reduced (resp. increased) if the last $\mu$ visited solutions are all feasible (resp. all infeasible), where μ is a user-defined parameter. The reduction (resp. increase) may consist in dividing (resp. multiplying) the actual value by 2, for example.



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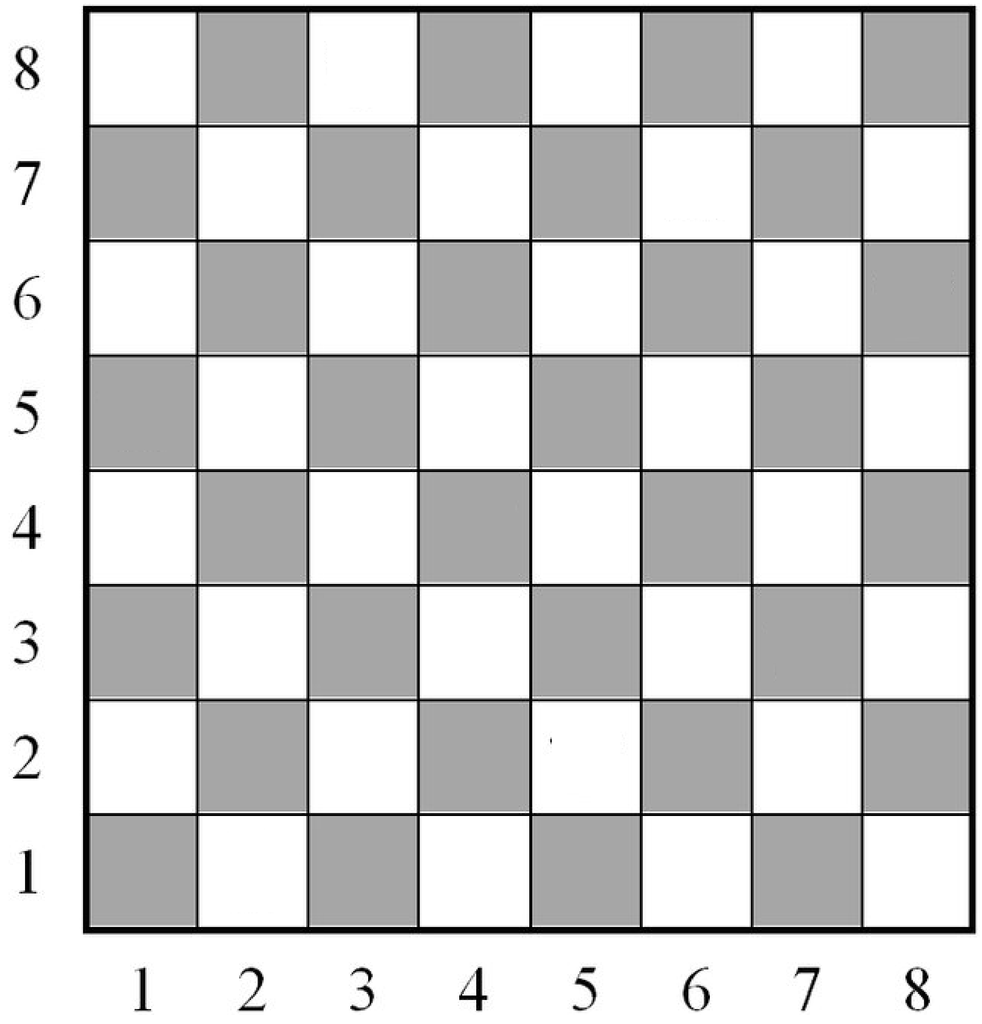
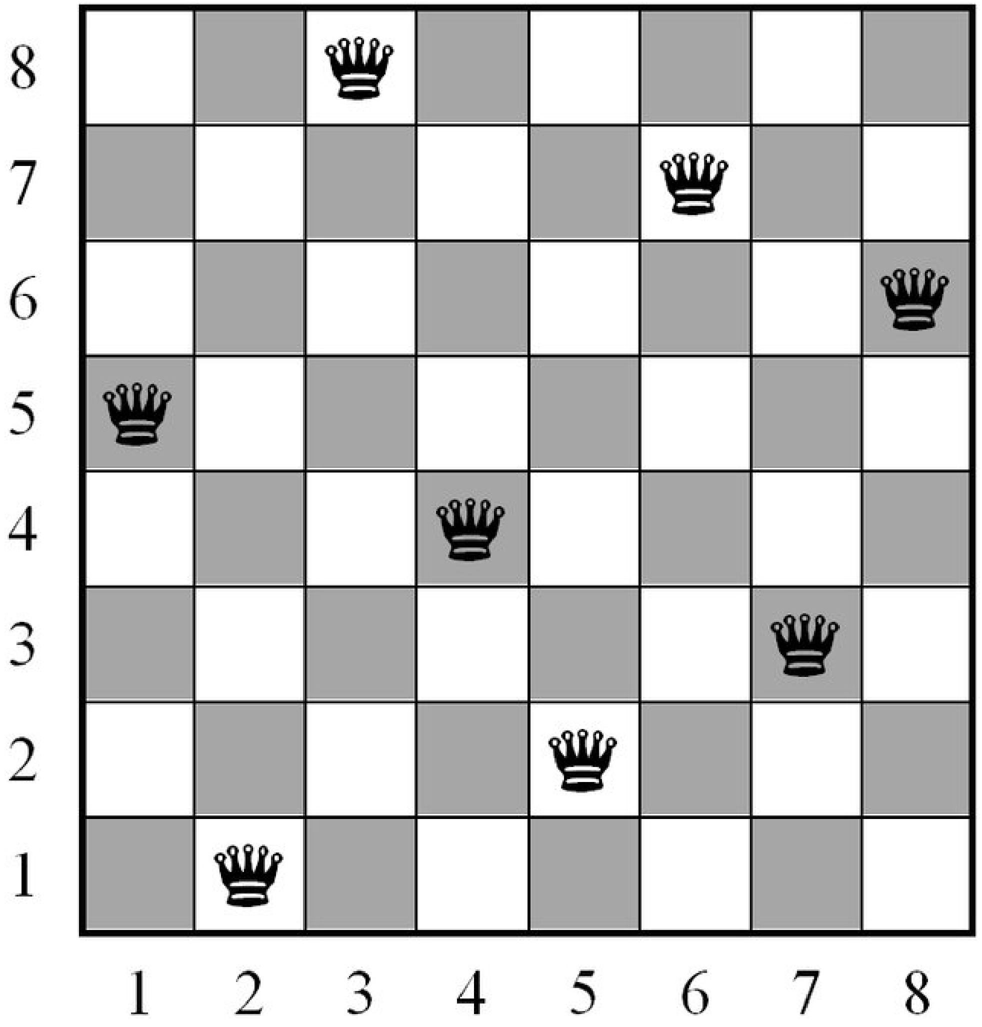
# Preserving Strategies

- Constructing a solution encoding/search operator that always guarantee that feasible solutions are obtained, is often not possible.
- Therefore, preserving strategies are not an alternative in many cases
- Yet, before applying other constraint handling techniques, one must think if it is possible to design a solution encoding/search operator that always provide feasible solutions
- Let’s take a look on a classic example: the N-Queens Puzzle



# Preserving Strategies

- The N-Queens Puzzle problem consists on putting N chess queens on an NxN chessboard such that none of queens is able to capture any other.
- By exhaustive search, the number of possibilities is $64^8$, that is over 4 billion solutions (size of the search space)
- If we prohibit more than one queen per row, then the search space will have $8^8$ solutions, that is over 16 millions.
- If we forbid two queens to be both in the same column or row, the encoding will be reduced to $n!$, that is 40,320



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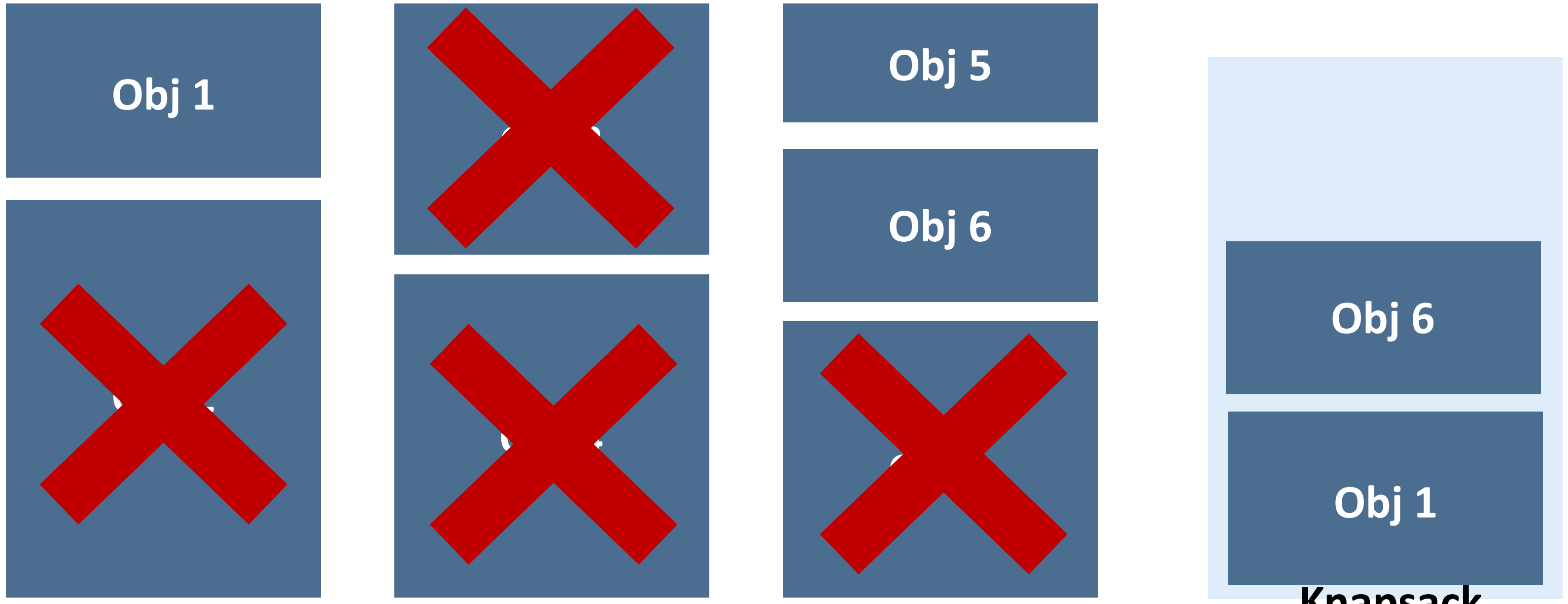
**Example Knapsack Problem:**

**Rejection Strategy**

**Objects**




**Knapsack**

**Solution Encoding**

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 0 | 0 |

**Search Operator**

Flip n bits of the solution

# Solution Encoding and Search Operator

- A solution encoding must have the following characteristics:
  - **Completeness**: all solutions associated with the problem must be represented.
  - **Connexity**: A search path must exist between any two solutions of the search space. Any solution of the search space, especially the global optimum solution, can be attained.
  - **Efficiency**: The representation must be easy to manipulate by the search operators. The time and space complexities of the operators dealing with the representation must be reduced.

**Solution encoding is key in metaheuristics performance – but also very problem specific!**

# Direct and Indirect Encoding

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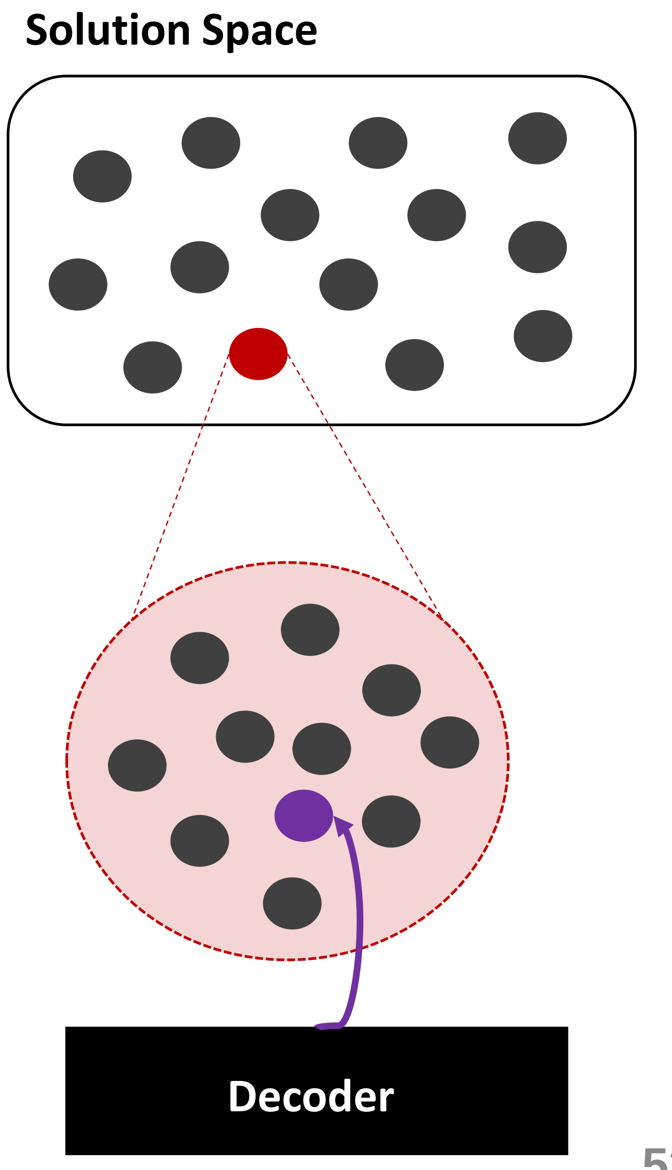
# Direct vs Indirect Encoding

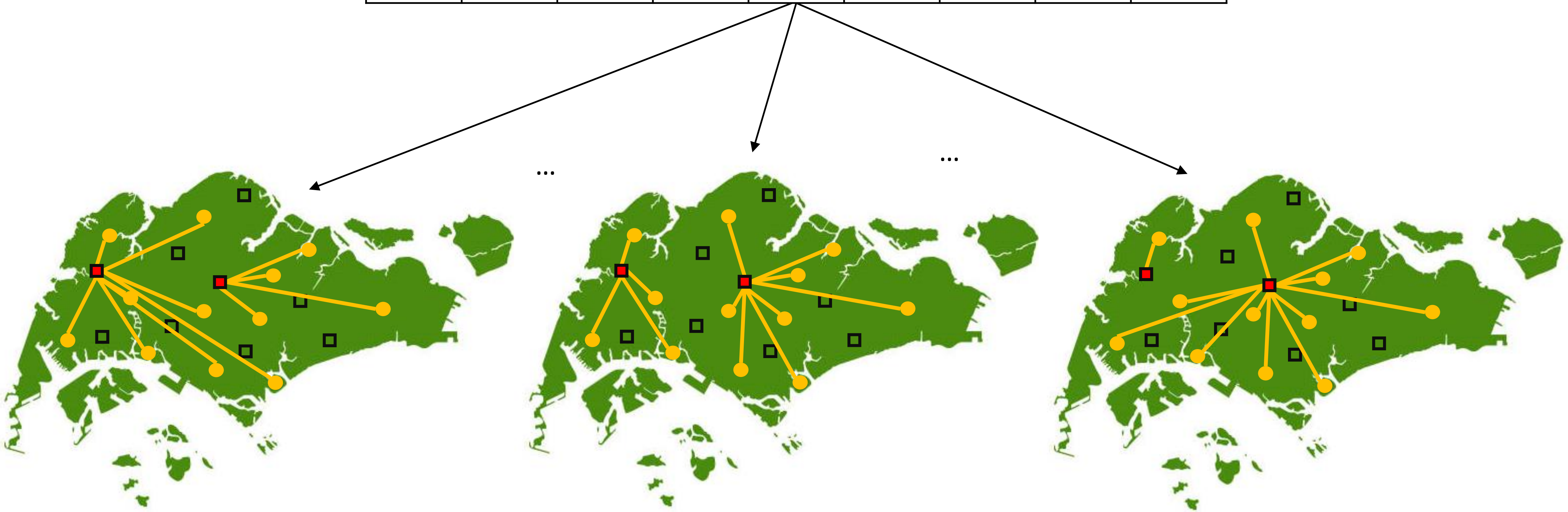
- **Indirect encoding** is characterized by a lack of details in the solution representation - i.e. some information on the solution is not explicitly represented.
- A **decoder** is required to express the solution given by the encoding. According to the information that is present in the indirect encoding, the decoder has more or less work to derive a complete solution
- Indirect encoding aims to **reduce the size of the original search space**. They are particularly popular in optimization problems **dealing with many constraints** such as scheduling problems.



# P-median Problem

Solution Encoding

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **1** | **0** | **0** | **1** | **0** | **0** | **0** | **0** | **0** |

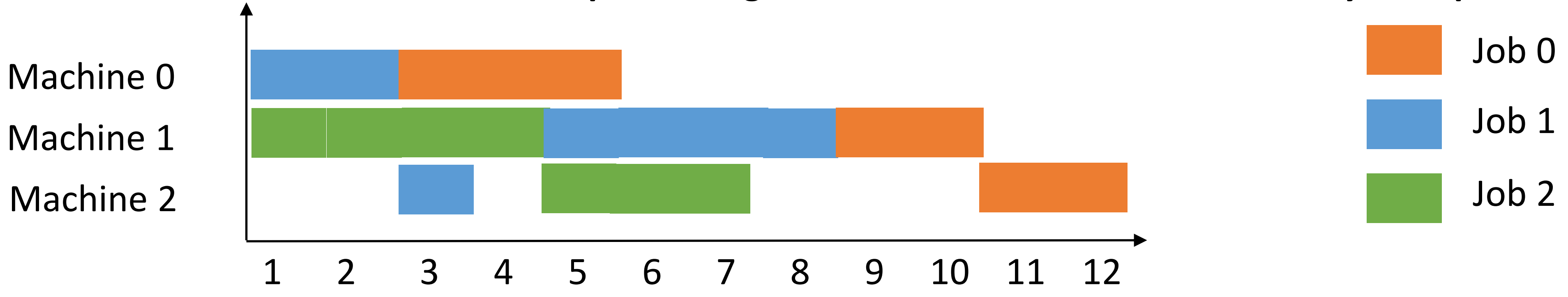


# Job-Shop Scheduling Problem (JSSP)

- In the classical Job-Shop Problem there are *n* jobs that must be processed on *m* machines. Each job consists of a sequence of different tasks. Each task needs to be processed during an uninterrupted period of time on a given machine.
- Here is an example with m=3 machines and n=3 jobs. We count jobs, machines and tasks starting from 0.

| Job | (Machine,Duration) | (Machine,Duration) | (Machine,Duration) |
|---|---|---|---|
| 0 | (0,3) | (1,2) | (2,2) |
| 1 | (0,2) | (2,1) | (1,4) |
| 2 | (1,4) | (2,3) | |

**Gant Chart representing a feasible solution - not necesseraly the optimal**



# JSSP Solution Representation

- **Direct** Encoding:
  - List of starting times:

| Job | (Machine, Duration) | (Machine, Duration) | (Machine, Duration) |
|---|---|---|---|
| **0** | (0,3) | (1,2) | (2,2) |
| **1** | (0,2) | (2,1) | (1,4) |
| **2** | (1,4) | (2,3) | |

| Job | Task | Task | Task |
|---|---|---|---|
| **0** | 1 | 2 | 3 |
| **1** | 4 | 5 | 6 |
| **2** | 7 | 8 | |

| Task 1 | Task 2 | Task 3 | Task 4 | Task 5 | Task 6 | Task 7 | Task 8 | Task 9 |
|---|---|---|---|---|---|---|---|---|
| 3 | 5 | 6 | 2 | 3 | 8 | 3 | 7 | 10 |

Very ineffective encoding ; Mostly infeasible solutions will be generated

# JSSP Solution Representation

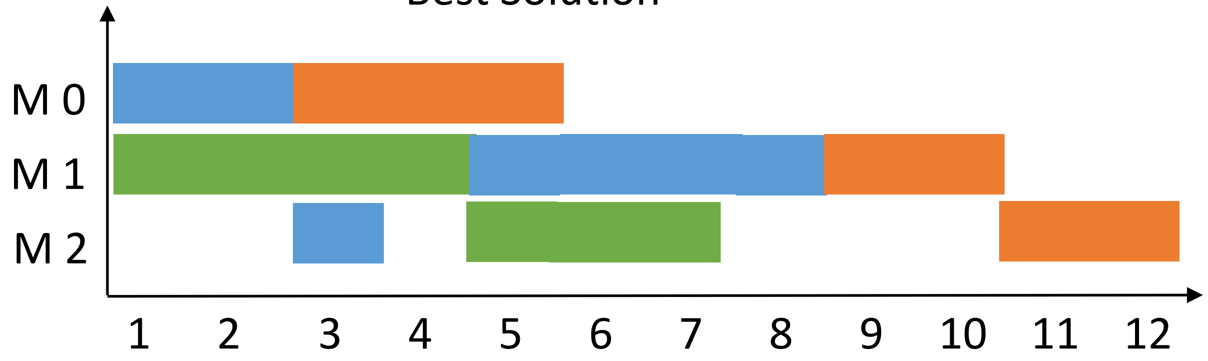
- **Indirect** Encoding:
  - Job Sequence Matrix:

| Machine 0 | | Machine 1 | | | Machine 2 | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 1 | 0 | 1 | 2 | 0 |

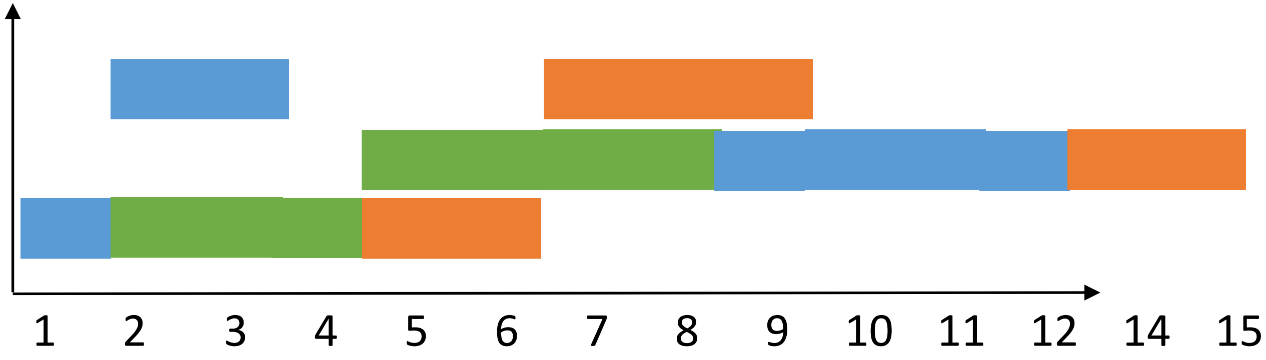
**In indirect encoding several solutions are represented by the same encoding. Some information on the solution is not explicitly represented. This will reduce the size of the original search space.**


Several solutions are represented by this encoding

Best Solution




Another solution that can be obtained using the same job sequence



# JSSP Solution Representation

- **How to obtain the best solution given a job sequence matrix?**
  - A Gant chart can be represented as a disjunctive graph G=(V,C,D).
    - V is the set of vertices corresponding to the tasks
    - C is a set of conjunctive arcs between tasks of a job
    - D is a set of disjunctive arcs between tasks to be processed on the same machine.
  - A **topological ordering algorithm** can be used to determine the optimal critical path of a gant chart (represented as a disjunctive graph)

| Job | (Machine, Duration) | (Machine, Duration) | (Machine, Duration) |
|---|---|---|---|
| 0 | (0,3) | (1,2) | (2,2) |
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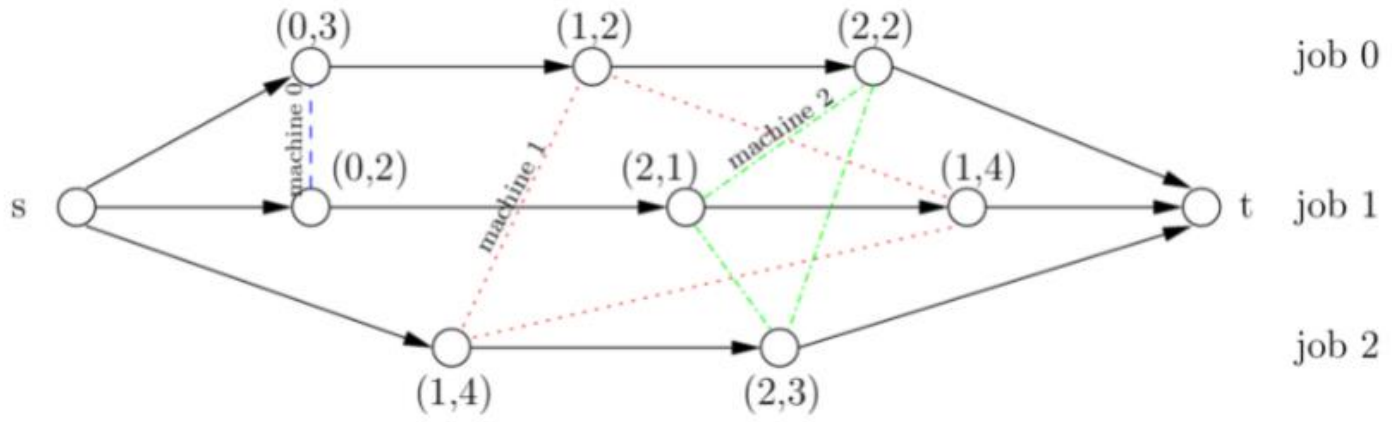
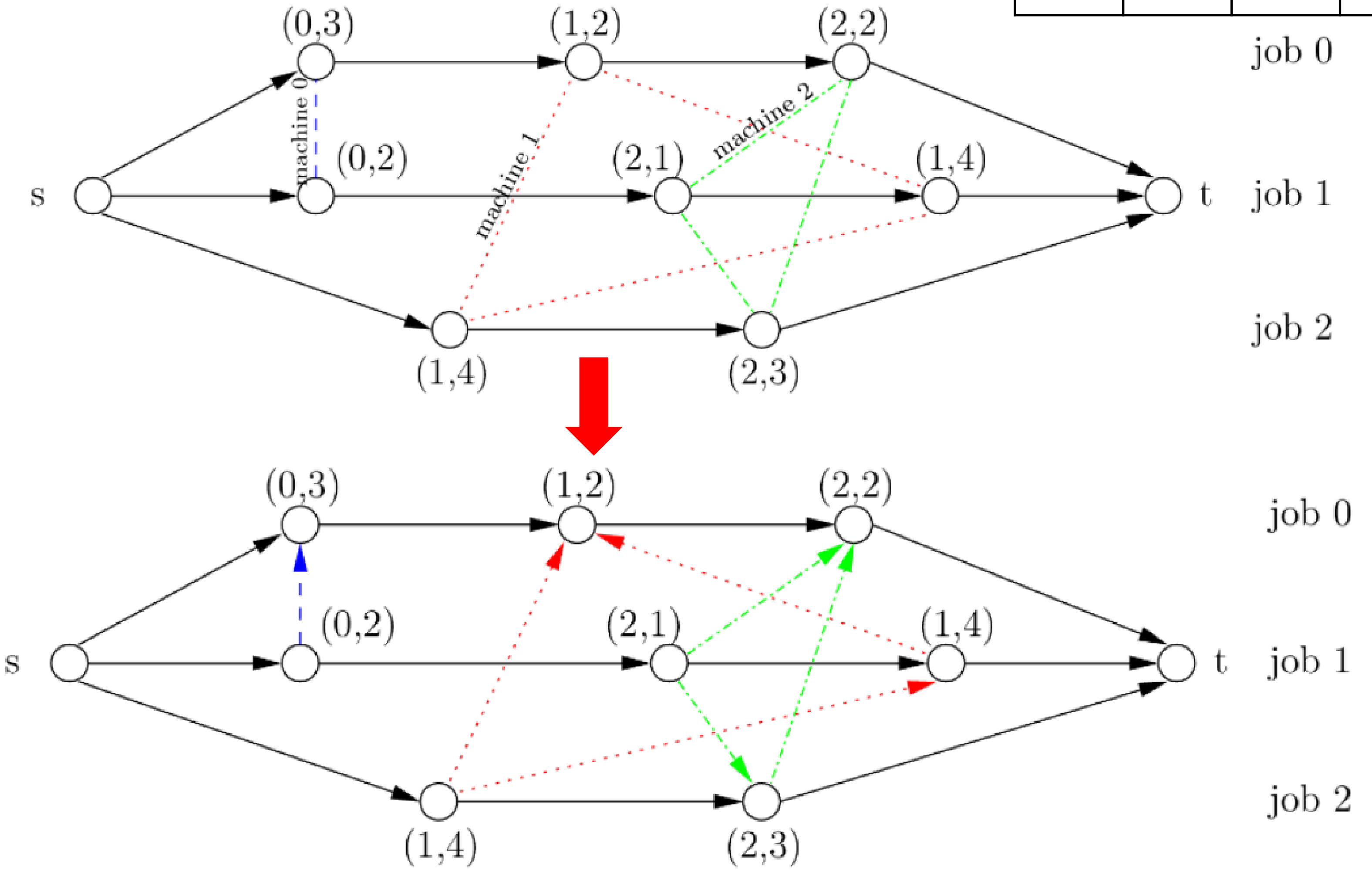


# JSSP Solution Representation

- Disjunctive graph solution representation

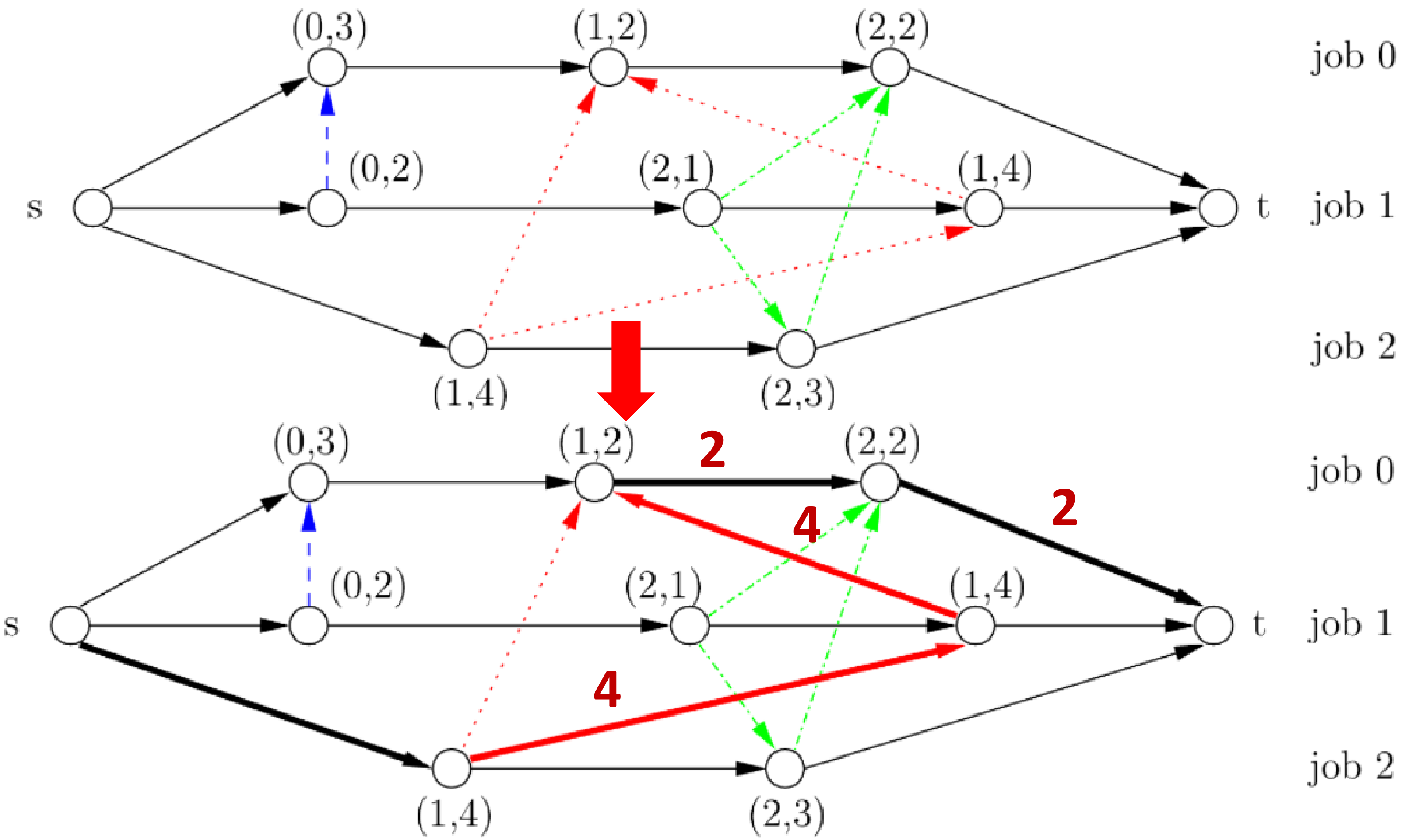
| Machine 0 | | Machine 1 | | | Machine 2 | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 1 | 0 | 1 | 2 | 0 |



# JSSP Solution Representation

- Optimal critical path is given by the longest weighted path from *s* to *t*

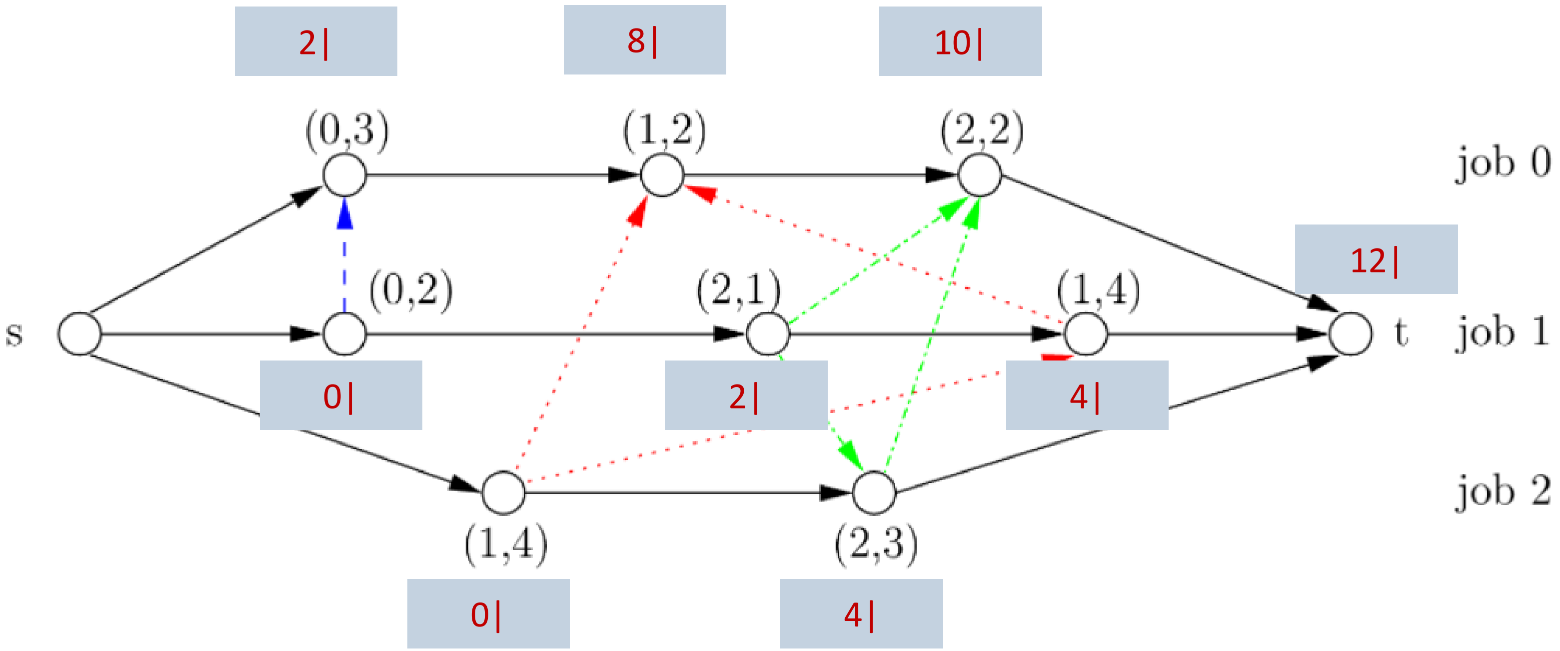
| Machine 0 | | Machine 1 | | | Machine 2 | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 1 | 0 | 1 | 2 | 0 |




**Critical Path Length =4+4+2+2=12**

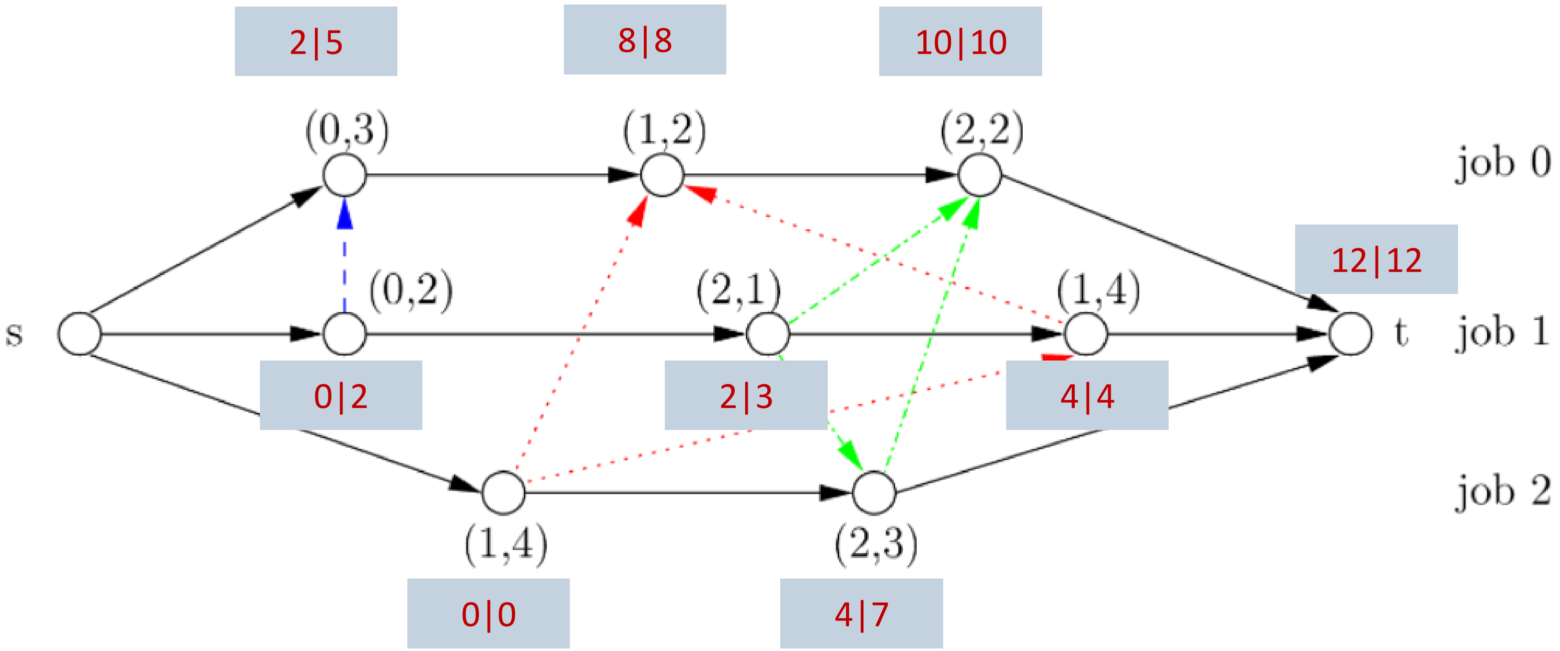
# JSSP Solution Representation

- How to obtain the longest weighted path of a disjunctive graph?
- We can use a topological ordering algorithm



# JSSP Solution Representation

- How to obtain the longest weighted path of a disjunctive graph?
- We can use a topological ordering algorithm

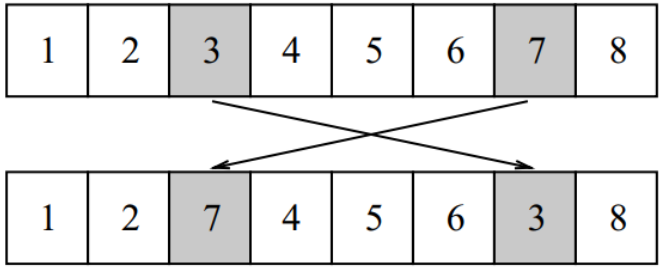


# JSSP Move Operators

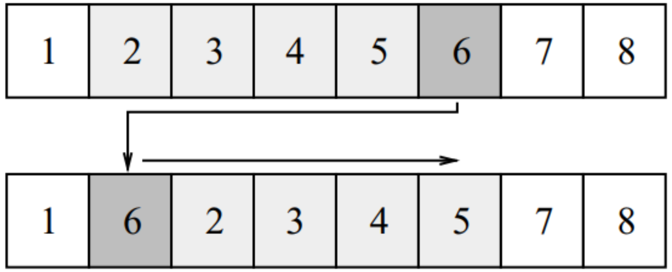
- **Permutation encoding**

| Machine 0 | | Machine 1 | | | Machine 2 | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 1 | 0 | 1 | 2 | 0 |

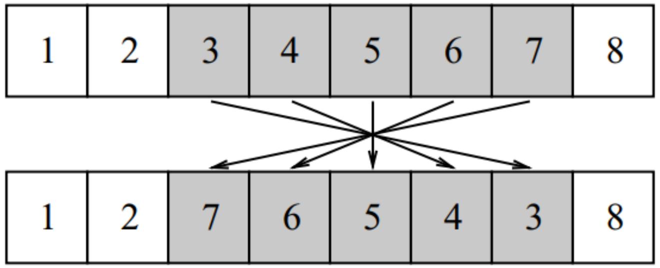
  - Swap/Exchange Operator




  - Insertion Operator

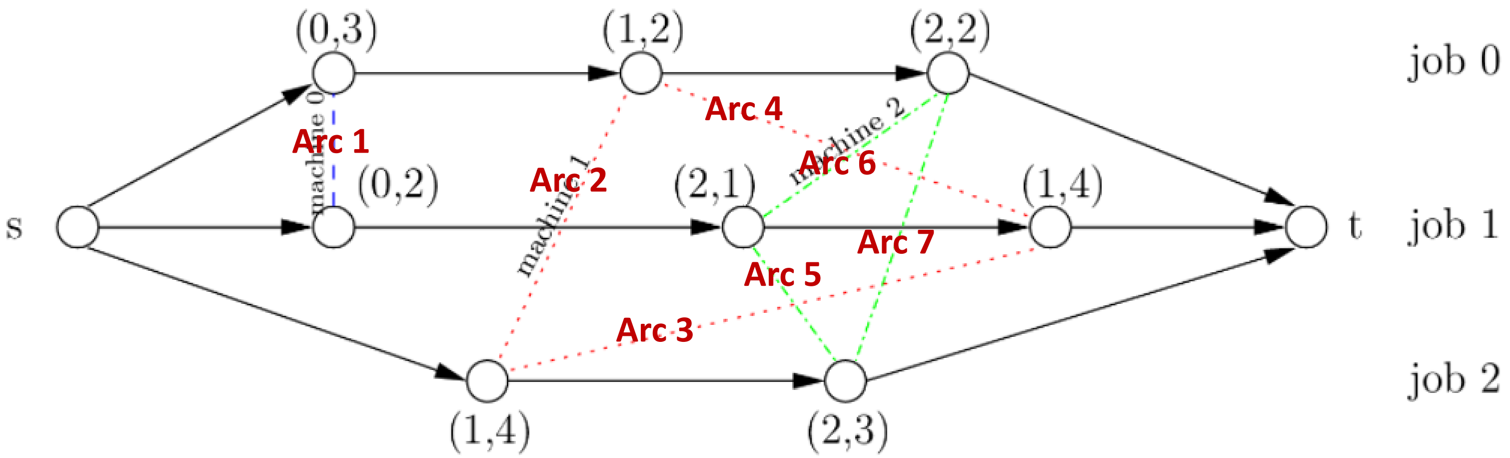



  - Inversion Operator



# JSSP Move Operators

- **Binary encoding** – each bit represents the orientation of a disjunction arc in the disjunction graph




| Arc 1 | Arc 2 | Arc 3 | Arc 4 | Arc 5 | Arc 6 | Arc 7 |
|---|---|---|---|---|---|---|
| 1 | 0 | **0** | 1 | 0 | 1 | 1 |

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| Arc 1 | Arc 2 | Arc 3 | Arc 4 | Arc 5 | Arc 6 | Arc 7 |
|---|---|---|---|---|---|---|
| 1 | 0 | **1** | 1 | 0 | 1 | 1 |

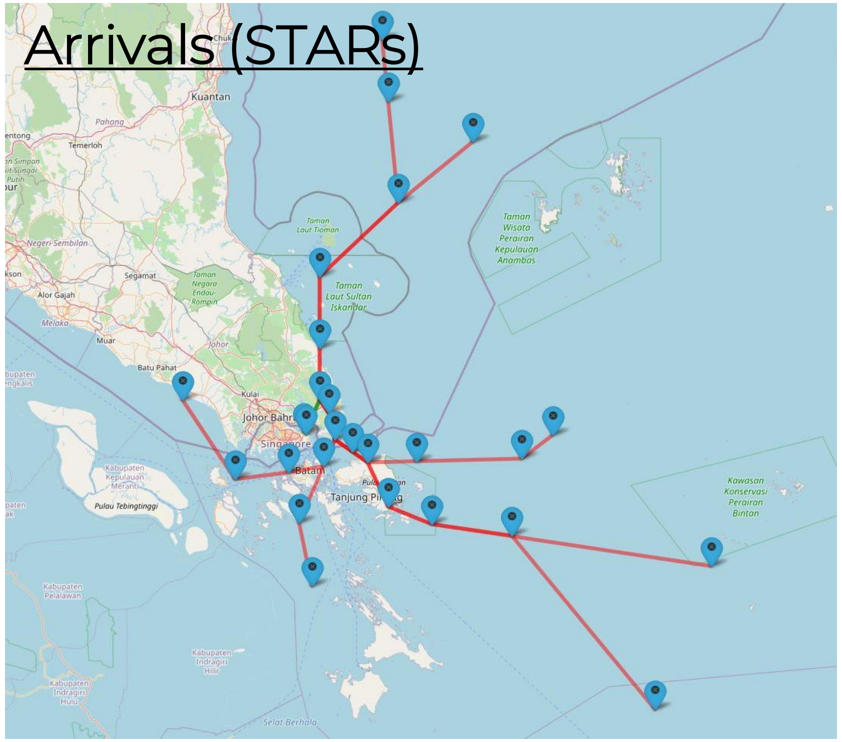
# Optimizing Terminal Maneouvering Airspace Operations

- TMA is designated area of controlled airspace surrounding a major airport where there is a high volume of traffic.
- It is a critical region - where all arriving aircraft from different entry points are merged and sequenced into an orderly stream towards the airport.
- Departing flights also use the TMA region, therefore air traffic controllers must ensure that arrivals and departures do not conflict with each other


**Radarbox website:** https://www.radarbox.com/

# SIDs and STARs

- Aircraft flying in the TMA must follow pre-defined routes, designated as Standard Terminal Arrival Routes (STAR) and Standard Instrument Departure (SID) routes.

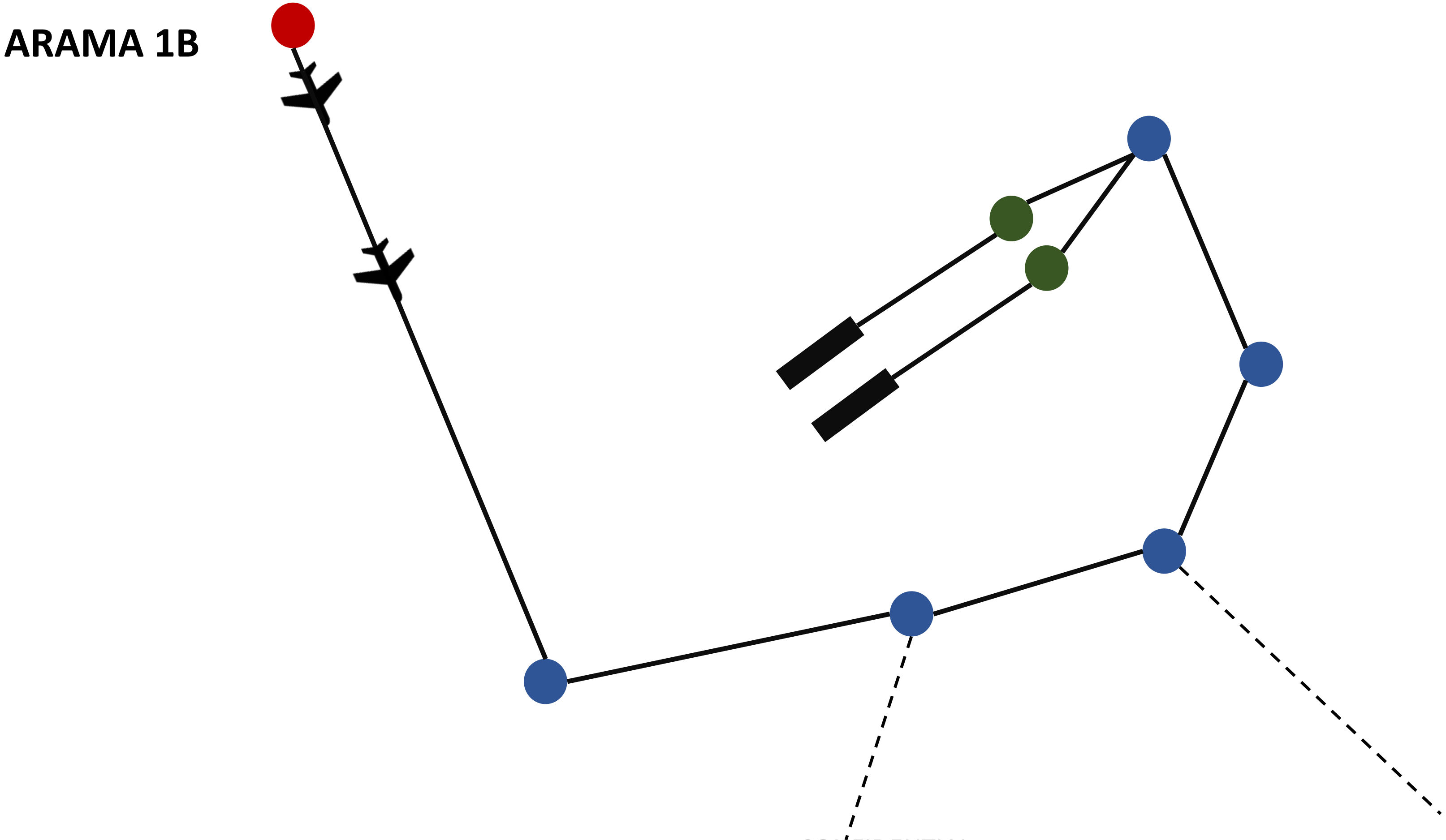




STAR - Standard Terminal Arrival Route
SID – Standard Instrument Departure

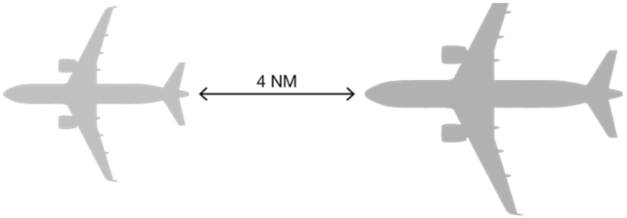
# TMA Operations

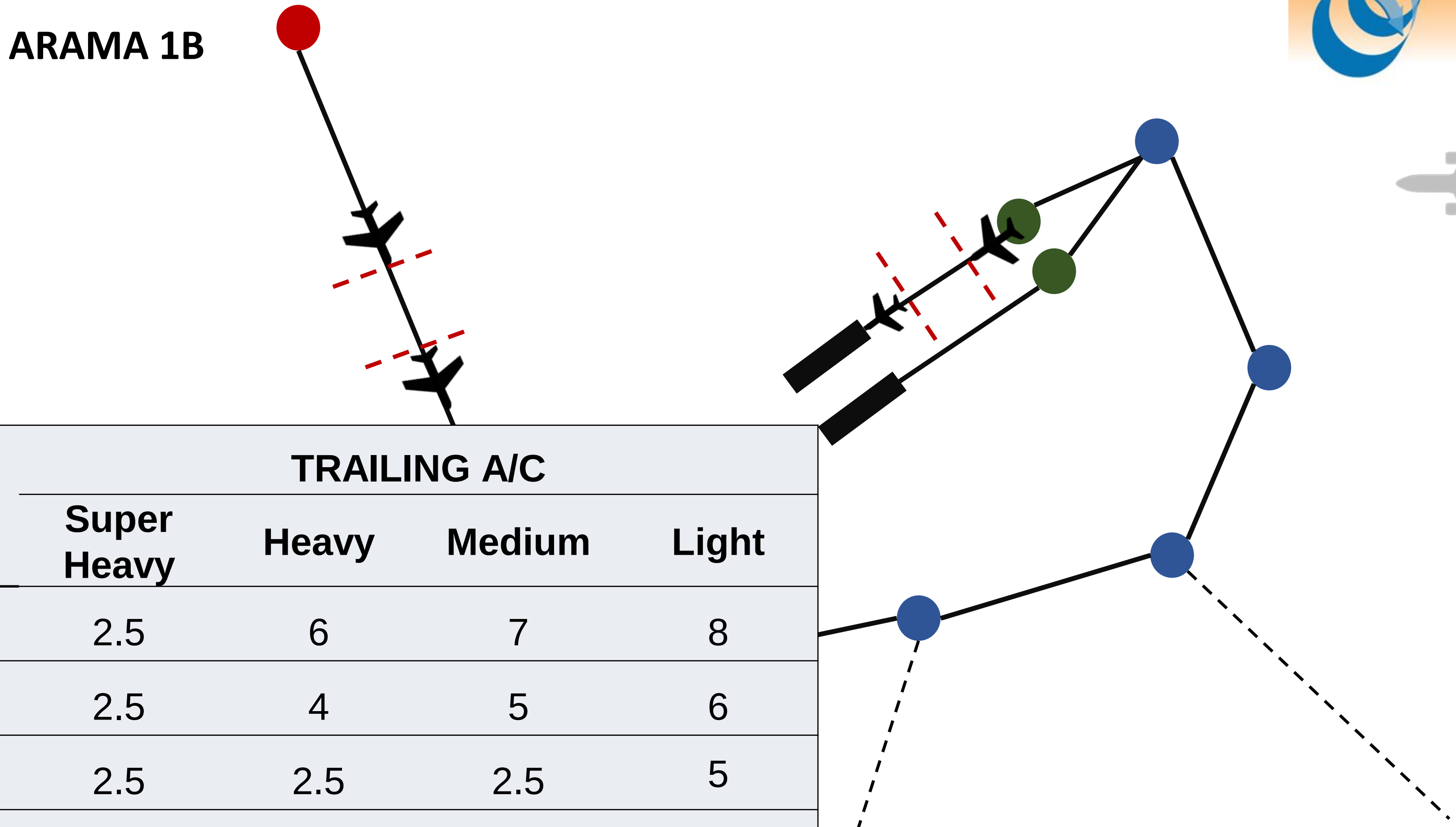
- Separation Requirements



# TMA Operations

- Optimization-based approach

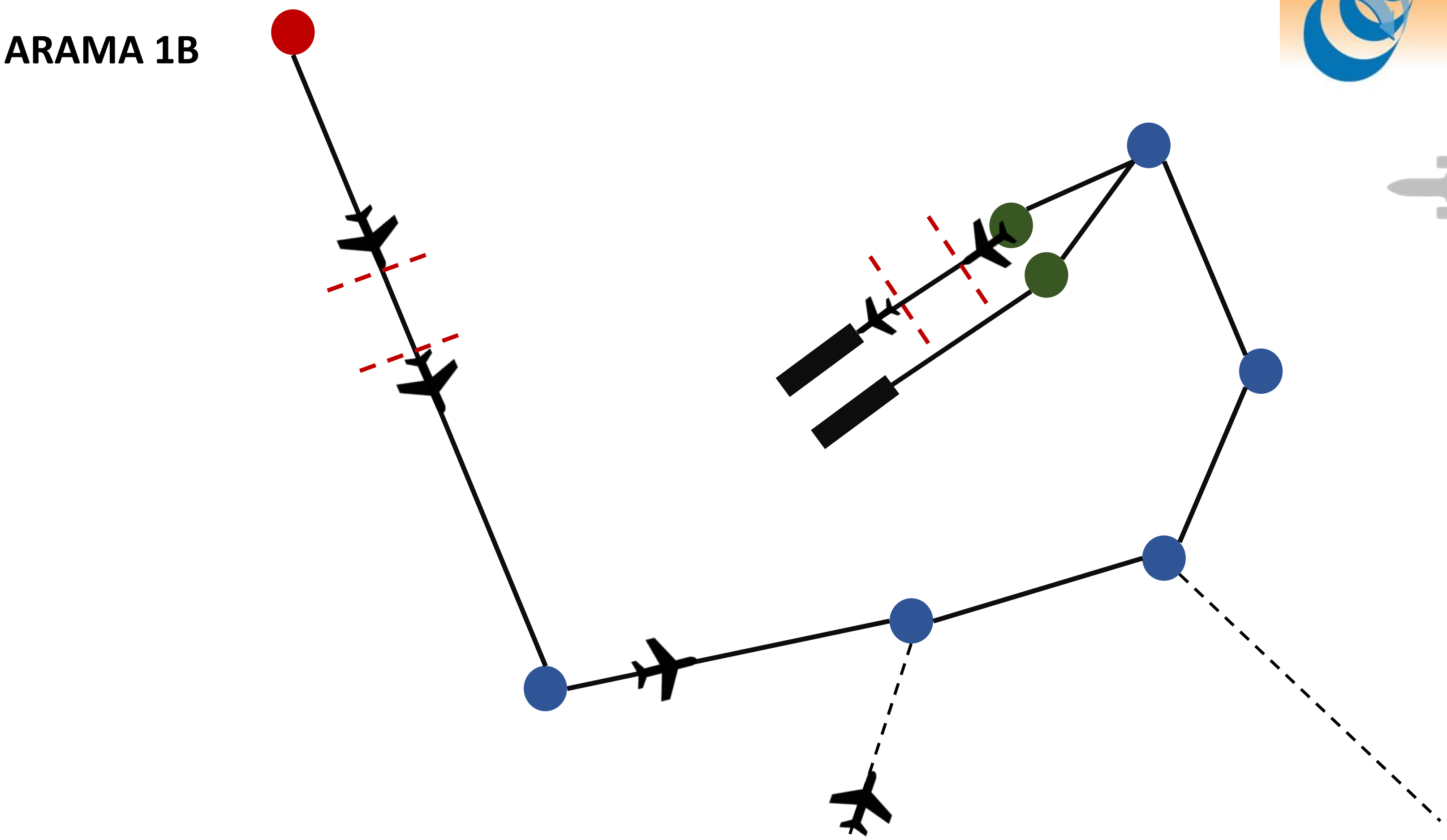


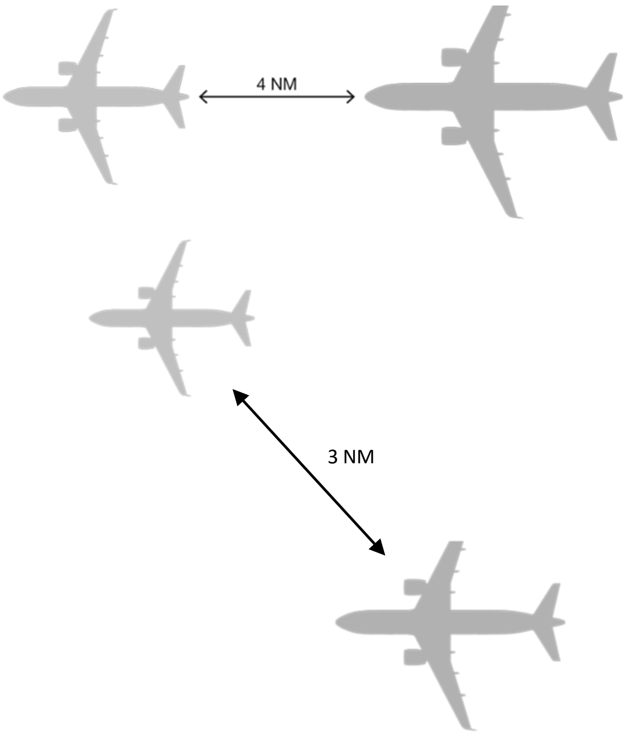




| LEADING A/C | TRAILING A/C | | | |
|---|---|---|---|---|
| | Super Heavy | Heavy | Medium | Light |
| Super Heavy | 2.5 | 6 | 7 | 8 |
| Heavy | 2.5 | 4 | 5 | 6 |
| Medium | 2.5 | 2.5 | 2.5 | 5 |
| Light | 2.5 | 2.5 | 2.5 | 2.5 |

# TMA Operations

- Optimization-based approach

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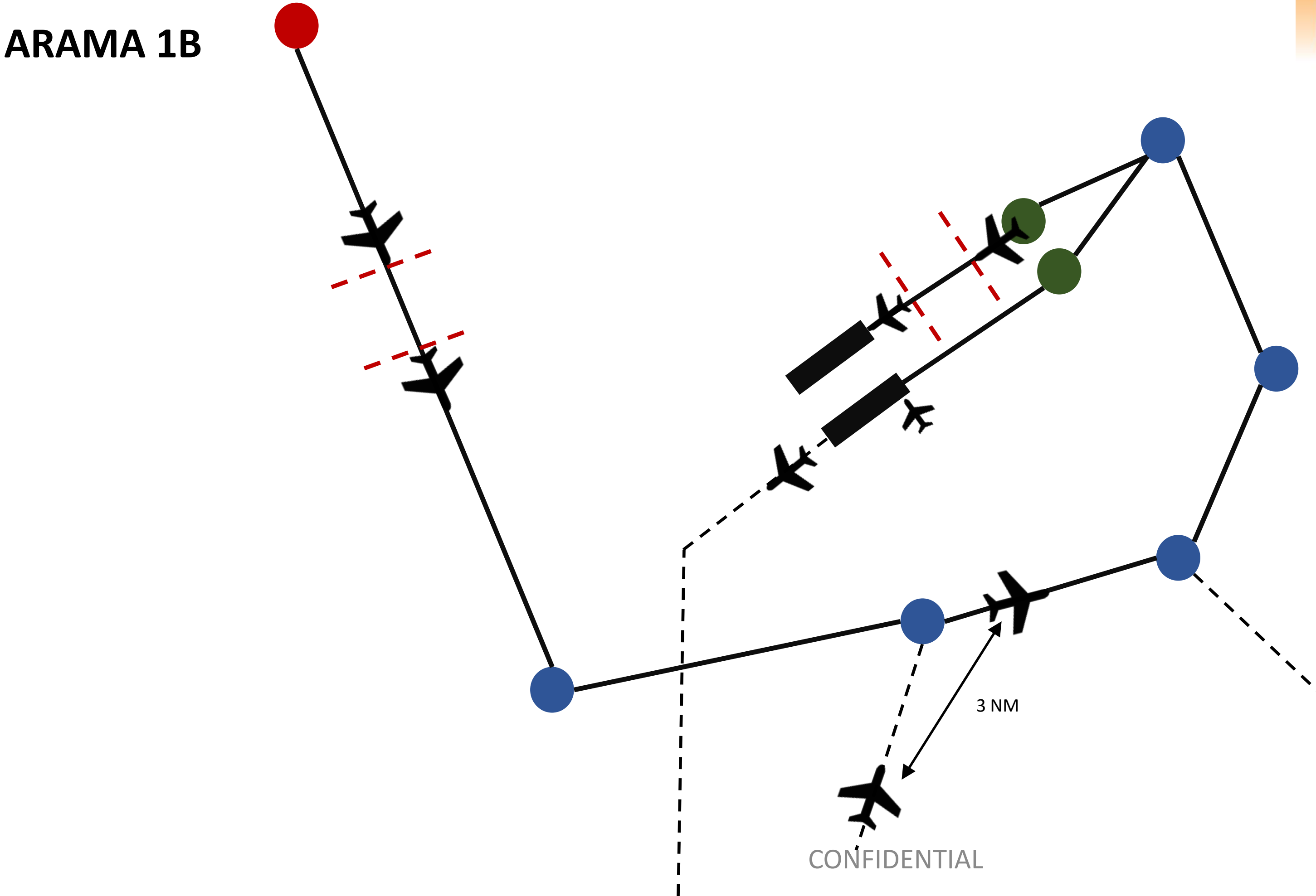
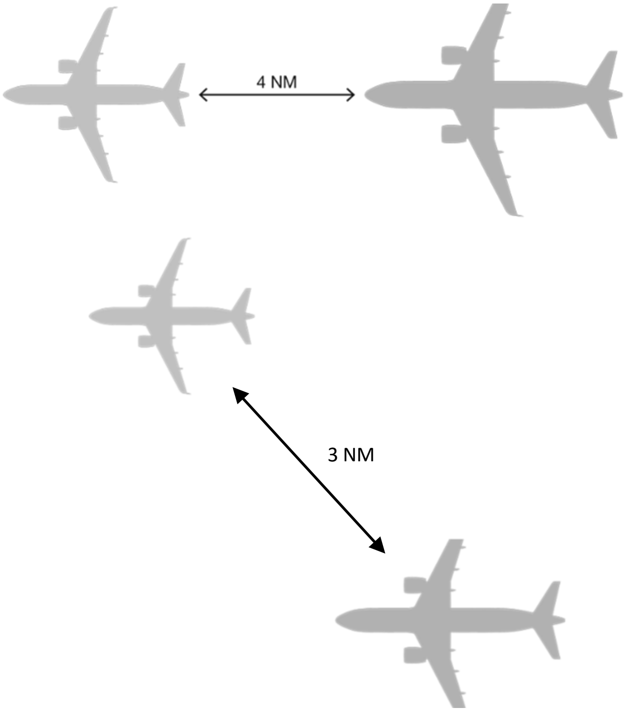
# TMA Operations

- Optimization-based approach

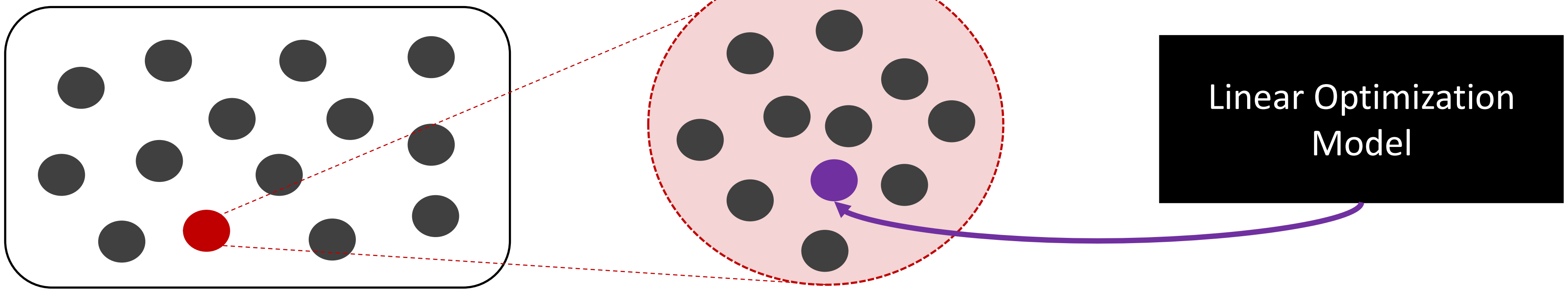



**Time-based Separation between Departures (e.g. 2 minutes)**

# Indirect Encoding

**Solution Space**




Solution representation: permutation encoding

| Aircraft Runway Sequencing | | | | | | | |
|---|---|---|---|---|---|---|---|
| Flights | 2 | 4 | 1 | 3 | … | 15 | 22 |

By fixing aircraft runway sequencing, aircraft speeds are optimized to minimize the amount of delays – the problem can be formulated as a linear optimization problem and can be solved in 30 seconds